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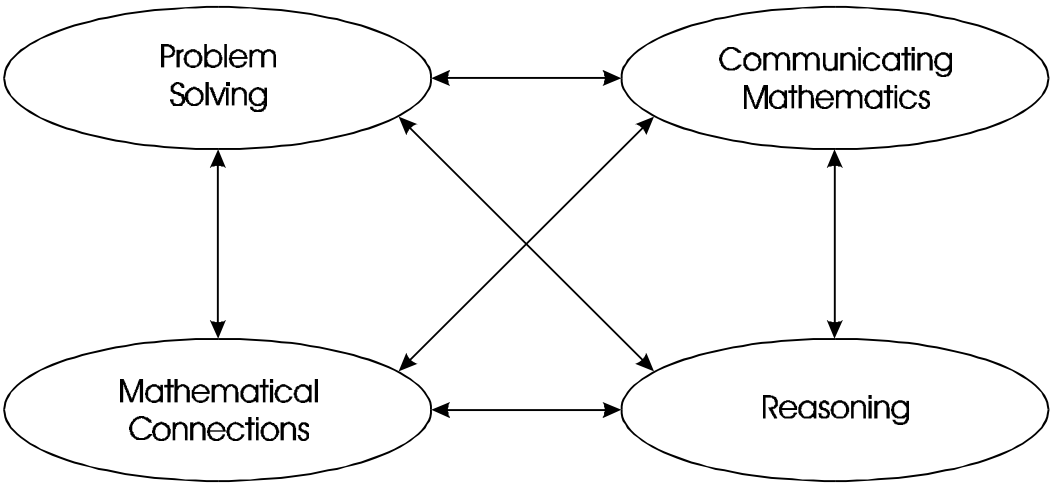
HOW CAN WE USE NJ DOE'S MATH CURRICULUM FRAMEWORKS TO IMPROVE KINDERGARTEN STUDENTS ACHIEVEMENT?

THE NEWARK TEACHERS UNION CAN HELP!

The First Four Standards — Grades K-2



Overview

Young children enter school with informal strategies for solving mathematical problems, communication skills, ideas about how number and shape connect to each other and to their world, and reasoning skills. In grades K-2, students should build upon these informal strategies.

Early instruction in **problem solving** should focus on taking time to understand the problem before rushing to solve it. Kindergartners should begin, for example, by representing problems using physical objects. By second grade, students should begin to move away from dependence on physical objects towards the use of pictures and figures. One of the goals of problem solving in numerical situations is to move students toward the use of more efficient problem solving strategies — from modeling with concrete objects to counting methods to using number facts. Even kindergartners should have experience with multiple-step problems (*Mary has 3 cookies. She eats one. Her mother gives her two more. How many cookies does she have now?*) in order to focus their attention on understanding the problem and developing a plan for its solution. Students should be able to describe how they have solved a problem and justify their answer. They should also develop the habit of comparing problems to each other, noting how they are alike and different.

Communication activities in grades K-2, whether with individuals, small groups, or the whole class, initially emphasize oral (e.g., counting) and pictorial representations. Much time is spent, however, in introducing students to symbolic representations (e.g., numerals and symbols for operations). As students develop written communication skills, they also begin to communicate in writing about mathematics. At first, the teacher may write the students' responses on the board or on sentence strips in order to facilitate this written communication. Students use many concrete representations (e.g., base ten blocks, pattern blocks) and need to learn how to represent their work with these manipulatives through pictures. Students also begin to communicate mathematics using graphs and diagrams.

Many **mathematical connections** begin to be established in kindergarten. Students should connect the number three to triangles, for example, as well as to sets of three objects and the numeral 3. Especially

important are quantification (*how much? how many?*), patterns, and representing quantities and shapes. Using children's literature to motivate and set a context for problem solving and learning mathematics is especially appropriate for K-2, as is illustrated in one of the following vignettes. Connections to social studies may involve using graphs to describe characteristics of the class, the school, or the community.

Many connections between science and mathematics can be established, from looking for patterns to developing specific skills in measurement and data collection. Children observe life cycles and cycles in nature, such as the seasons, and the growth and decay of plant forms. Children begin by using words to describe physical characteristics: color intensity (bright or dull), sound volume (loud or quiet), temperature (hot or cold), and size (longest or shortest). This allows them to make simple descriptive comparisons and to place objects in an order. They move on to using numbers to describe such characteristics. For example, students might measure the height of plants at different times, summarize their data in a table, and prepare a graph (bar or line) showing the height over time. They might repeat the experiment with different growing conditions, and then compare their graphs for the different conditions.

Students in grades K-2 should spend a great deal of time on inductive **reasoning**, looking for patterns, making educated guesses, generating hypotheses, and forming generalizations based on their experiences. They should also begin to develop some skill in drawing logical conclusions and justifying answers (deductive reasoning), perhaps by using manipulatives such as attribute blocks. They should continually strive to make sense of mathematics by using reasoning to predict answers and compare and contrast examples and problem situations.

In grades K-2, students build on what they already know as they develop their skills in problem solving, communication, mathematical connections, and reasoning. They begin to move from informal, intuitive strategies and processes towards more symbolic representations and more explicit recognition of their thinking strategies.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

The First Four Standards — Grades K-2

Vignette — Will a Dinosaur Fit?

Standards: In addition to the First Four Standards, this vignette highlights Standards 6 (Number Sense), 7 (Geometry), 9 (Measurement), and 11 (Estimation).

The problem: The second grade was in the midst of a unit on dinosaurs when the teacher read to her class the book *Danny and the Dinosaur* by Syd Hoff (Harper & Row, 1958). After the first reading, the children re-examined some of the illustrations. One picture depicted the dinosaur larger than a block of homes, another showed the dinosaur almost completely hidden by one house. One picture showed the dinosaur taller than an apartment building and yet another showed the dinosaur not quite as tall as a lamp post. Students were intrigued by the idea that Danny's dinosaur friend did not seem to be of a consistent size. They voiced opinions about the dinosaur's actual size. Since students seemed to have a sustained interest in exploring the sizes of dinosaurs, the teacher presented students with this question: *Do you think that a dinosaur could fit into our classroom?*

The discussion: Brainstorming was encouraged by the teacher as questions such as the following were posed by students and by the teacher. *What does it mean to "fit" in the classroom? What information would we need to get in order to determine if a dinosaur could fit in our classroom? Do you think all of our answers will be the same? Why? What do we know already that might help us? What materials do you think we would need?*

Solving the problem: Students worked in groups of 3 over a period of several days. They began by choosing a specific dinosaur and then they used a variety of books and computer software in the classroom to find the size of their dinosaur. They determined the size of the classroom, choosing to measure with a trundle wheel or a tape, or by using estimation. Then they decided, by comparing the measures found in books with those made of the classroom, whether the dinosaur would fit into the classroom. Each group was responsible for creating a display and making a presentation to the class to answer the question. The displays made use of models, pictures, and text. Students with more than a few sentences to write were encouraged to make use of the word processor available in the classroom.

Summary: Students used their displays to make presentations to the class. There were a variety of answers. Those who had chosen one of the smaller dinosaurs, the velociraptors, for example, found that the dinosaur could walk through the doorway and several dinosaurs would fit in the room. Others, who had chosen larger dinosaurs, the stegosaurus, for example, found that if the dinosaur could have gotten through the doorway, several would have fit in the room. Still others, who had chosen very large dinosaurs, the brachiosaurus, for example, found that the dinosaur would not have fit into the room at all. As the presentations ended, several children suggested further explorations that might be interesting: *Would the dinosaur I chose fit into the multi-purpose room? Was the dinosaur I chose as long as the driveway in front of the school? Was the dinosaur I chose taller than the school building?*

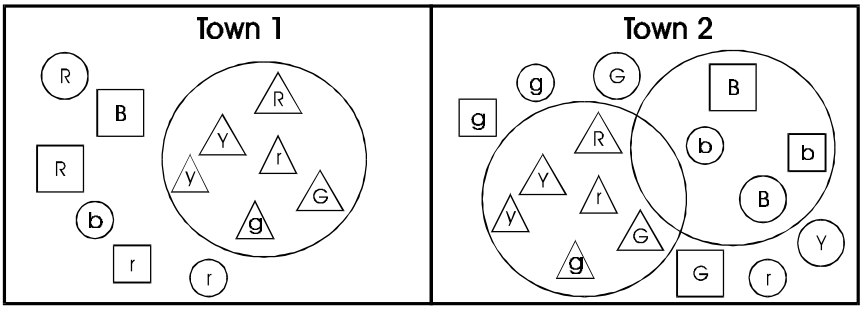
The First Four Standards — Grades K-2

Vignette — Shapetown

Standards: In addition to the First Four Standards, this vignette highlights Standards 7 (Geometry), 11 (Patterns), and 14 (Discrete Mathematics).

The problem: The students in kindergarten had been involved in a unit that allowed them to explore their town. They had been exposed to a variety of activities, including building symmetric and non-symmetric block buildings, drawing neighborhood maps, and using letter-number ordered pairs (like A-2) to locate places on a grid. In this lesson, pairs of students were challenged to build towns with attribute blocks and loops based on a rule or pattern that they made up.

The discussion: With the class sitting on the carpet in a circle, the teacher placed a loop within everyone's sight. She explained that the loop was a town and that the blocks were buildings. Using blocks of different colors, she then placed several triangles inside the loop and several non-triangles outside the loop. Ideas about the rule used to build Town 1 were discussed: *Tell me about the town. Describe a pattern that you see. Put this triangle on the carpet to follow the pattern. Put this circle on the carpet to follow the pattern. How could you tell someone else about our town so they could build one just like it?* The verbalization was then called *the rule* for the town. Town 2 was created with two loops, blocks were placed inside and outside these loops, and similar questions were raised and discussed. Several reasonable rules were suggested. For example, one rule was: triangles in one loop, blue blocks in the other loop, other colors and shapes in the overlapping loop and outside the loop. Another rule was: triangles in one loop, blue blocks in the other loop, blue triangles in the overlapping loop, and all other blocks outside the loops.



Solving the problem: Students were given loops and some attribute blocks. They were challenged to work together to build a town that used a rule. At the end of the working time, each pair of students challenged the class to place other blocks in their town and then to verbalize the rule that was used to create the town.

Summary: Students worked independently to record their town designs using crayons and shapes cut from colored construction paper. Students described the rules that they used to build their towns.

The First Four Standards — Grades K-2

Indicators

The cumulative progress indicators for grade 4 for each of the First Four Standards appear in boldface type below the standard. Each indicator is followed by a brief discussion of how the preceding grade-level vignettes might address the indicator in the classroom in kindergarten and in grades 1 and 2. The Introduction to this *Framework* contains three vignettes describing lessons for grades K-4 which also illustrate the indicators for the First Four Standards; these are entitled *Elevens Alive!*, *Product and Process*, and *Sharing a Snack*.

> Standard 1. All students will develop the ability to pose and solve mathematical problems in mathematics, other disciplines, and everyday experiences.

Experiences will be such that all students in grades K-2:

1. **Use discovery-oriented, inquiry-based, and problem-centered approaches to investigate and understand mathematical content appropriate to the early elementary grades.**
    - *Will a Dinosaur Fit?* uses the question *Do you think a dinosaur would fit into our classroom?* to launch an investigation involving measurement, geometry, estimation, and large numbers. *Shapetown* develops students' logical (deductive) reasoning skills using shapes (geometry), sorting (discrete mathematics), and pattern analysis.

2. **Recognize, formulate, and solve problems arising from mathematical situations and everyday experiences.**
    - In *Will a Dinosaur Fit?*, students recognize and help to formulate the question they will investigate, based on a book they have read and its illustrations. In *Shapetown*, students develop their own logic problems in connection with a unit in social studies on their community.

3. **Construct and use concrete, pictorial, symbolic, and graphical models to represent problem situations.**
    - Students in *Will a Dinosaur Fit?* begin with a pictorial model (the pictures in the book) and then use numerical models and graphs to represent the problem situation. Students in *Shapetown* use concrete materials (attribute blocks) to represent their problem situation and then record their "rules" using pictures.

4. **Pose, explore, and solve a variety of problems, including non-routine problems and open-ended problems with several solutions and/or solution strategies.**
    - In *Will a Dinosaur Fit?*, each group investigates a different dinosaur, using their own strategies. Different groups have different answers, depending on the size of their dinosaur.

In *Shapetown*, pairs of students pose their own problems for the others to solve.

5. **Construct, explain, justify, and apply a variety of problem-solving strategies in both cooperative and independent learning environments.**

    - The students in *Will a Dinosaur Fit?* work in groups of three, measuring the classroom and collecting data from books. The students in *Shapetown* work in pairs to develop the rules for their towns.

6. **Verify the correctness and reasonableness of results and interpret them in the context of the problems being solved.**

    - Students in *Will a Dinosaur Fit?* present their results to the class for verification. The students in *Shapetown* verify their results by having other students solve their problems.

7. **Know when to select and how to use grade-appropriate mathematical tools and methods (including manipulatives, calculators and computers, as well as mental math and paper-and-pencil techniques) as a natural and routine part of the problem-solving process.**

    - In *Will a Dinosaur Fit?* students select their own measuring tools and some use computers. They decide whether to estimate or measure and how to determine their answers (compare numbers or subtract mentally or with a calculator or with paper-and-pencil). The students in *Shapetown* use manipulatives (attribute blocks) to develop their rules.

8. **Determine, collect, organize, and analyze data needed to solve problems.**

    - The students in *Will a Dinosaur Fit?* determine what information they need to know about their dinosaurs, collect that information, organize it and analyze it. The students in *Shapetown* organize and analyze the placement of objects in the town in accordance with the rules they were given and the rules they generated or discovered.

9. **Recognize that there may be multiple ways to solve a problem.**

    - In their sharing, the students in *Will a Dinosaur Fit?* find out about the many different ways in which students address this problem. The students in *Shapetown* might explain how they figure out the "rules" their classmates use for their own towns.

Standard 2. All students will communicate mathematically through written, oral, symbolic, and visual forms of expression.

Experiences will be such that all student in grades K-2:

1. **Discuss, listen, represent, read, and write as vital activities in their learning and use of mathematics.**

    - In *Will a Dinosaur Fit?,* the students read a story, read information from books about their dinosaurs, represent their results using symbols and words, and explain their results orally. In *Shapetown*, the students listen to the teacher explain how to develop a "rule," discuss

their rules in pairs as they develop them, and record their rules with a picture.

2. **Identify and explain key mathematical concepts, and model situations using oral, written, concrete, pictorial, and graphical methods.**

    - The students in *Will a Dinosaur Fit?* model their problem situations using oral and written language. Some groups may also use pictorial and/or graphical methods. The students in *Shapetown* use concrete materials to model their problems and oral methods to solve them.

3. **Represent and communicate mathematical ideas through the use of learning tools such as calculators, computers, and manipulatives.**

    - Some students in *Will a Dinosaur Fit?* use computers; others use trundle wheels or measuring tape. Students in *Shapetown* use manipulatives (attribute blocks).

4. **Engage in mathematical brainstorming and discussions by asking questions, making conjectures, and suggesting strategies for solving problems.**

    - The teacher in *Will a Dinosaur Fit?* begins the discussion of the problem by having students brainstorm what it means for a dinosaur to fit in the classroom. The students in *Shapetown* discuss the problems posed by the teacher and make conjectures as they try to solve them.

5. **Explain their own mathematical work to others, and justify their reasoning and conclusions.**

    - Students in *Will a Dinosaur Fit?* explain their work and justify their reasoning about their group's dinosaur. Students in *Shapetown* explain their work and justify their results as they challenge each other to solve their problem.

Standard 3. All students will connect mathematics to other learning by understanding the interrelationships of mathematical ideas and the roles that mathematics and mathematical modeling play in other disciplines and in life.

Experiences will be such that all students in grades K-2:

1. **View mathematics as an integrated whole rather than as a series of disconnected topics and rules**.

    - In both vignettes, the students are investigating problems that involve several content standards.

2. **Relate mathematical procedures to their underlying concepts.**

    - In *Will a Dinosaur Fit?*, students research the size of their dinosaurs, determine the size of their classroom by measuring, and compare the measures to see which is larger. In *Shapetown*, students apply the fundamental concepts of Venn diagrams.

3. **Use models, calculators, and other mathematical tools to demonstrate the connections among various equivalent graphical, concrete, and verbal representations of mathematical concepts.**

    - In *Will a Dinosaur Fit?*, students create a display and make a presentation to the class to support their conclusion. In *Shapetown*, the students verbalize the rule used for their town and then create an equivalent representation for their attribute block models using a picture.

4. **Explore problems and describe and confirm results using various representations.**

    - The second-graders in *Will a Dinosaur Fit?* use a variety of representations (symbols and words) to record their results as they investigate the problem. The students in *Shapetown* use a pictorial representation to describe their results.

5. **Use one mathematical idea to extend understanding of another.**

    - The teacher in *Will a Dinosaur Fit?* uses the students' understanding of relative size to extend their understanding of estimation and measurement. The students in *Shapetown* use their understanding of geometric shapes to build their "rules" as they learn more about logical reasoning.

6. **Recognize the connections between mathematics and other disciplines, and apply mathematical thinking and problem solving in those areas.**

    - The dinosaur lesson involves applying mathematics to learn about dinosaurs (science). The *Shapetown* lesson builds upon a social studies unit in which students use mathematics to locate buildings, construct buildings, and draw maps.

7. **Recognize the role of mathematics in their daily lives and in society.**

    - The students in *Will a Dinosaur Fit?* learn how mathematics is involved in the sizes of illustrations in the books that they read. The *Shapetown* students learn how mathematics is used in buildings, in determining locations, and in classifying and characterizing objects.

Standard 4. All students will develop reasoning ability and will become self-reliant, independent mathematical thinkers.

Experiences will be such that all students in grades K-2:

1. **Make educated guesses and test them for correctness.**

    - The students in *Will a Dinosaur Fit?* could address this indicator by predicting whether their dinosaur will fit before measuring the classroom. The students in *Shapetown* are challenged to guess the rule for placing blocks on the carpet, and then to verbalize the rule they think is being used.

2. **Draw logical conclusions and make generalizations.**

    - The students in *Will a Dinosaur Fit?* draw conclusions from the data they collect by

measuring and using texts or the computer. They might also make some generalizations about dinosaurs collectively after discussing the results of all the groups. Drawing logical conclusions is the major focus of the *Shapetown* lesson.

3. **Use models, known facts, properties, and relationships to explain their thinking**.

   - The students in *Will a Dinosaur Fit?* use models, known facts (from books and software), and relationships to explain how they know whether their dinosaur will fit. The *Shapetown* students use models to explain their thinking.

4. **Justify answers and solution processes in a variety of problems.**

   - Students in both vignettes justify their answers and solution processes.

5. **Analyze mathematical situations by recognizing and using patterns and relationships.**

   - The students in *Will a Dinosaur Fit?* solve their problems by comparing the sizes of the various dinosaurs with other sizes, such as the classroom and its doorway. The students in *Shapetown* recognize and use patterns and relationships as they pose and solve their problems involving attribute blocks.

Standard 5 — Tools and Technology — Grades K-2

Overview

This standard addresses the use of calculators, computers and manipulatives in the teaching and learning of mathematics. These tools of mathematics can and should play a vital role in the development of mathematical thought in students of all ages.

In the primary grades, **manipulatives** are the most natural of the three types of tools to use. Primary grade teachers have traditionally used many manipulative materials in their teaching of mathematics because they correctly perceived them to be of great value for young children. Typically, concrete materials are used to model mathematical concepts such as number or shape when those concepts are first introduced to the students.

Young children counting with lima beans, colored chips, linking cubes, smooth stones, or their fingers is a familiar sight in many New Jersey classrooms as they begin to master early counting skills and are introduced to addition and subtraction concepts. More sophisticated models should then be used, though, to begin to explore more sophisticated number concepts. Colored rods in graduated lengths give students a different sense of number than a set of discrete objects. Students should be able to see both a yellow rod **and** five colored chips as representative of the number five — the first being more of a measure model and the second a count model. Ice cream sticks and base ten blocks as well as chip trading activities help students begin to understand the very abstract concepts involved with place value and number base.

Attribute blocks, blocks with different shapes and colors, help students begin to classify and categorize objects and recognize their specific characteristics. Pattern blocks allow them to make patterns and geometric designs as they become familiar with the geometric properties of the shapes themselves. Geoboards allow students to explore the great variety of shapes that can be made and also to deal with issues of properties, attribute, and classification.

A great variety of different materials should be used to explore measurement. Paper clips, shoes, centimeter and decimeter rods, paper cutouts of handspans, and building blocks can all be used as non-standard units of length (even though some of them are really standard). Students place them down one after another to see how many paper clips long the desk is or how many handspans wide the doorway is. The transition can then be made to more standard measures and, following that, to rulers.

This list is, of course, not intended to be exhaustive. Many more suggestions for materials to use and ways to use them are given in the other sections of this *Framework.* The message in this section is a very simple one — concrete materials help children to construct mathematics that is meaningful to them.

Calculators have not been used traditionally in primary classrooms, but there are several appropriate uses for them. It is never too early for students to be introduced to the tool that most of the adults around them use whenever they deal with mathematics. In fact, many students now come to kindergarten having already played with a calculator at home or somewhere else. To ignore calculators completely at this level is to send the harmful message that the mathematics being done at school is different from the mathematics being done at home or at the grocery store.

The use of calculators at this level does not imply that students don't need to develop the arithmetic skills traditionally introduced at the primary level. They certainly do need to develop these skills. This Standard does not suggest that all traditional learning be replaced by calculator use; rather, it calls for the appropriate and effective use of calculators.

One of the most effective uses of the calculator with young children is the use of the constant feature of most calculators to count, forward or backward, or to *skip count*, forward or backward, by twos or threes or other numbers. This process allows children to anticipate what number will come next and then get confirmation of their guess when they see it appear in the display. Students can also greatly enhance their estimation ability through calculator use. *Range-finding games* ask students, for instance, to add a number to 34 that will give them an answer between 80 and 90. After the estimate is made, it is punched into the calculator to see whether or not it did the job. Calculators will prompt young students to be curious about mathematical topics that are not typically taught at their level. For example, when counting back by threes by entering 15 – 3 = = = . . . into the calculator, after the expected sequence of 12, 9, 6, 3, 0, the child will see –3, –6, –9, . . . A curious child will begin to ask questions about what those numbers are, but will also begin to develop an intuitive notion about negative numbers.

Computers are a valuable tool for primary children. As more and more computers find their way into primary classrooms, the software available for them will dramatically improve; however, there are already many good programs that can be used with kindergartners and first and second graders. *MathKeys* links on-screen manipulative materials to standard symbolic representations and to a writing tool for children to use. A number of different counting programs match objects on the screen to a standard symbolic representation of the number and the number is said aloud so that a young student can count along with the program. Many other new programs focus on money skills and help children recognize different coins and determine the values of sets of coins through simulated purchases.

Standard 5 — Tools and Technology — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Select and use calculators, software, manipulatives, and other tools based on their utility and limitations and on the problem situation.**
    - Students participate in races to complete a set of computation problems between some students who use calculators and others who use mental math. They try to determine what makes the calculator a useful tool in some circumstances (large numbers, harder operations) and not terribly useful in others (basic facts, easy numbers).
    - Students are regularly asked to make their own decisions about what is the right type of linear measuring device for a particular situation: mental estimation, colored rods, ruler, yard or meter stick, or tape measure. Different decisions are made in different circumstances: Estimation is fine when you are deciding whether you will fit through a small doorway, but accurate ruler measurement is important if you are cutting out a frame for a picture.
    - In problem solving situations, students are regularly provided with calculators, manipulatives, and other tools so that they may choose for themselves what will be useful to help solve the problem.

2. **Use physical objects and manipulatives to model problem situations, and to develop and explain mathematical concepts involving number, space, and data.**
    - Students use popsicle sticks to model multi-digit base-ten numbers and then use them to further model operations with the numbers.
    - Students use pipe cleaners and straws to make models of two-dimensional geometric shapes. They then compare, contrast, and sort all the shapes using whatever criteria they think are important, including number of corners, straight or curvy sides, number of sides, and so on.
    - Students work through the *Shapetown* lesson that is described in the First Four Standards of this *Framework.* Students in kindergarten are challenged to build towns with attribute blocks and loops based on a rule or pattern they make up.
    - Kindergarten students each use a cubic inch block to represent himself or herself in a bar graph that describes the favorite flavors of ice cream of all the students in the class. On a table in the front of the room, the teacher has placed mats that say *Vanilla, Chocolate,* and *Strawberry*. One by one, the students walk past the table, dropping their blocks on one of the piles that build up on the mats. When this concrete "bar graph" is complete, the children ask questions that can be answered with the data displayed: *What's the most favorite flavor in the class? What's the least favorite? Are there more people who like vanilla than*

chocolate?

3. **Use a variety of technologies to discover number patterns, demonstrate number sense, and visualize geometric objects and concepts.**

    - Students use the constant function on a calculator to count by ones, twos, tens, fourteens, and other numbers, both forward and backward. As they do so, they try to keep up with the calculator by saying the numbers orally as they come up in the display, and even trying to say them before they come up.
    - Students use a beginner's Logo to explore movement in two-dimensional space. They move the turtle on the computer screen forward and backward with simple commands and also turn the turtle through predetermined angles to the right and to the left with other commands. The turtle leaves a trail of where it's been on the screen so that its movements actually create a drawing of a figure. The students try to have the turtle draw a square, a different rectangle, and a triangle before progressing to harder tasks.
    - Students use a geoboard to make shapes that are composed of unit squares. One challenge they are given is to find as many shapes as possible that are made up of 10 unit squares.

4. **Use a variety of tools to measure mathematical and physical objects in the world around them.**

    - Young students develop meaning for rulers by first measuring with individual paper clips, then a paper clip chain, then taping the clip chain to a paper strip, then marking and numbering the ends of the clips on the strip, and last, removing the clip chain from the paper strip leaving just the marks and the numbers. This leaves the students with *paper clip rulers* with which they can measure the lengths of a variety of objects. The unit of measurement is, of course, a paper clip.
    - Students use a balance scale to determine the weights of a variety of classroom objects in terms of units that are other classroom objects; for example, *How many pennies does a math book weigh? How many paper clips does a pencil weigh?*
    - Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework.* Second grade students measure the size of their classroom and other places in a variety of ways to determine whether dinosaurs they are studying would fit into them.
    - As part of the morning calendar routine, second graders check each of two thermometers — one Fahrenheit and one Celsius — and make daily recordings of the outside temperature. They record the temperatures in a chart and look for interesting patterns. They notice that, as the school year progresses and the temperatures change, whenever one of the temperatures goes up or down, so does the other.
    - Students regularly use both analog and digital stopwatches to practice timing events that are usually measured in seconds such as: the amount of time it takes a classmate to say the alphabet, how long a classmate goes without blinking, or how long the morning announcements take.

5. **Use technology to gather, analyze, and display mathematical data and information.**

- Students take a survey to determine every child's birth month and then use the *Graph Club* or *Primary Graphing and Probability Workshop* software to display the resulting data in graphs.
- Using a World Wide Web page that reports meteorological data (possibly http://www.rainorshine.com/weather/index/sites/njo/), students find the predicted high temperatures for a variety of cities in different regions around the country, write those numbers on a map of the United States, and then look for patterns and trends in different regions.
- Students use *Table Top* software to make a Venn diagram to show which of them have brothers, which have sisters and which have both (the intersection of the two sets). Students who have no siblings are shown outside the rings. Other attributes of the children are also used to make Venn diagrams.

References

Software

Graph Club. Tom Snyder Productions.

Logo. Many versions of Logo are commercially available.

Primary Graphing and Probability Workshop. Scott Foresman.

TableTop. TERC.

MathKeys. Minnesota Educational Computing Consortium (MECC).

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards.*

Standard 6 — Number Sense — Grades K-2

Overview

Students can develop a clear sense of number from consistent ongoing experiences in classroom activities where a variety of manipulatives and technology are used. The key components of number sense, as identified in the K-12 Overview, include an **awareness of numbers and their uses** in the world around us, a good sense of **place value concepts, approximation, estimation, and magnitude**, the **concept of numeration**, and an understanding of **comparisons** and the **equivalence** of different representations and forms of numbers.

Kindergarten, first, and second graders are just beginning to develop their concepts of number. They have most likely come to school with some ability to count, but with differing notions of what that activity means. It is in these grades that they begin to attach meaning to the numbers that they hear about and see all around them. One useful activity that can be repeated many times throughout this age range is the keeping of a scrapbook reflecting all the **uses of numbers** that the children can identify. It would probably include telephone numbers, addresses, ages, page numbers, clothing sizes, room numbers, and many others. Discussions of the similarities and differences in all of these uses can provide some interesting insights.

In terms of **numeration**, students in these grades start by constructing meaning for one-digit numbers and build up to formal work with three-digit numbers. The regular and consistent use of concrete models for that development is essential. Kindergartners need a variety of things to count, from poker chips to marbles to beans. Both concrete and rote counting are critically important in developing a sense of number. Adequate attention to counting activities throughout these grades will help to assure both a good sense of **magnitude (size) of numbers** and a real readiness for all four basic operations. (See Standard 8.) Counting by ones should be followed by counting back; skip counting by twos, fives, and tens; counting from a given starting number to a given target number by ones and by other numbers; counting on by tens from non-multiples of ten like 43; and so on.

As students are able to handle larger numbers, place value and base-ten ideas are introduced through grouping activities. Many of the models with which they are comfortable for single units can translate nicely into beginning base-ten models; poker chips can be put in groups of ten into small paper cups; beans can be pasted in tens onto tongue depressors, and so on. These newly enhanced models, along with the single digit units, are then used to represent two-digit numbers. As the next step, of course, groups of ten tens can be made to create hundreds. These first models of base-ten number are the best ones to use with young children who are first encountering these notions because they can actually build larger units from smaller units. Such models are called *bundle-able*. Another property these have is *proportionality*, because the model for a ten is actually ten times as large as the model for a one. A widely used model which is both bundle-able and proportional involves popsicle sticks which are wrapped into tens and hundreds with rubber bands.

The next type of model to be used would be one which is still proportional, but no longer bundle-able. The best examples of this type are the standard base-ten blocks. They require the child to trade ten ones for a ten rather than directly constructing a ten from the ones, and, as a result, are slightly more sophisticated. The last level of sophistication in this sequence of models includes those that are neither proportional nor bundle-able.

Two models of this type which are regularly used are chip trading materials and play money. With chip trading materials, there is no inherent concrete ten-to-one relationship between the red chips and the green chips; the red chips are not ten times as large as the green ones. The relationship holds only because of an external rule that is made up and followed. Similarly, there is no inherent concrete ten-to-one relationship that exists between dimes and pennies. The relationship only exists because of a rule that is external to the coins themselves. As a result, these most sophisticated models should be used *after* the underlying concepts are developed with the earlier models.

Children at these grade levels also begin to learn about **equivalence**. When youngsters find as many "names" as they can for the number *7* (such as *2 + 5*, *9 – 2*, and *one more than 6*), they are creating equivalent forms of the same number. Slightly older students should be using similar activities to generate equivalent forms of multi-digit numbers, partly in preparation for operations involving them: *67 = 6 tens and 7 ones = 5 tens and 17 ones = 4 tens and 27 ones*.

Estimation should be a routine part, not only of daily mathematics lessons, but also of the entire school day. Children should be regularly engaged in estimating both quantities and the results of operations. They should respond to questions that arise naturally during the course of the day, like: *About how many kids do you think there are out here in the playground? About how many pieces of construction paper will we need for this project if everyone needs three different colors?* and *How many of your great graphs do you think will fit on the bulletin board without overlapping?* After several children have had chances to make estimates about numbers like these, they should defend their estimates by giving some rationale for thinking they are close to the actual number. These discussions can be invaluable in helping them to develop good number sense.

Technology plays an important role in number sense at these grade levels. Calculators can be wonderful teaching tools when programmed to count forward and backward by some constant. Children can do the programming easily themselves and try to anticipate the calculator display. Appropriate computer software provides environments in which students can first develop a sense of small whole numbers and then build an understanding of place-value and base-ten ideas.

The topics that should comprise the number sense focus of the kindergarten through second grade mathematics program are:

whole number meanings through three digits
place value and number base
counting and grouping

Standard 6 — Number Sense — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Use real-life experiences, physical materials, and technology to construct meanings for whole numbers, commonly used fractions, and decimals.**
    - Young students make and use a variety of models for "number" ranging from poker chips to dot patterns on a paper plate, to Cuisenaire Rods, to tally marks, to domino and dice combinations. A large component of their early work with number focuses on the various *parts* into which any given number can be broken.
    - Students play the *Broken Key* game on their calculators. Kindergartners try to get the calculator display to show *7* while pretending that the *7* key is broken and cannot be pressed. Second graders might try to get the display to show *45* without pressing the *4* or the *5* key.
    - Students use *5-frames* and *10-frames* to help develop initial ideas of small numbers. By filling up a 5-cell grid with counters first and then putting out 2 more while trying to show "7 in all," the child not only learns about "7" but also about its relationship to "5."
    - Students use numbers throughout the school day as they discuss the date, attendance, time, snacks, money, etc.
    - Students investigate fractions by listening to the story *Gator Pie* by Louise Mathews and by discussing how Alvin and Alice can share their pie with more and more alligators.
    - Second-graders record prices as decimals ($0.39) and use this notation to find totals over $1 on a calculator.
    - Students find half of a sheet of paper by folding horizontally, by folding vertically, and by folding diagonally. They compare the results and discuss how they are alike and how they are different.
    - Students use *Balancing Bear* software to find combinations of numbered weights that will balance a seesaw or that will be greater or less than a given weight.

2. **Develop an understanding of place value concepts and numeration in relationship to counting and grouping.**
    - Calendar activities at the beginning of the school day incorporate a *Daily Count* feature where each day another popsicle stick is added to a collection representing all of the days of school to date. Whenever 10 single sticks are available, they are bundled with a rubber band and are thereafter counted as a *ten*. On the hundredth day of school, the ten *tens* are wrapped together to make a *hundred*, and the class celebrates the event with a party.

- Students progress from a *proportional* and *bundle-able* base ten model like popsicle sticks to a *proportional* but not *bundle-able* model like base-ten blocks to a model that is neither *proportional* nor *bundle-able* like pennies and dimes. (See K-2 Overview)
- Pairs of students play *Race to One Hundred* with base ten blocks. Each, in turn, rolls one or two dice and takes that many unit cubes. Whenever there are ten unit cubes in a player's collection, the player *must* trade for a ten block. The first player able to trade ten ten blocks for a hundred block is the winner.
- Students have 3 dimes and 4 pennies to spend on a variety of items that are displayed in a classroom store. The items have tags ranging from 3¢ to 56¢ and the children are asked: *Which of these items can be bought for exactly the amount of money that you have (requiring no change)? Which items can you buy and have some money left over? Which of these items cannot be bought because you do not have enough money? What items are left?*
- Student understanding of place value for two-digit numbers is assessed by asking each student to represent a different number using popsicle sticks or base 10 blocks.

3. See patterns in number sequences, and use pattern-based thinking to understand extensions of the number system.

- Students find patterns in a hundred number chart. When asked to describe patterns that they see, some children see a counting by ones pattern horizontally, others see the tens digit increasing and the ones digit staying the same as they move down the chart vertically, and still others see in the last column the numbers that they use to count by tens.
- Students use the constant function feature of their calculators to program a *skip count*. They press *+ 2 ===* to watch the display count by twos, try to anticipate what number comes next and make predictions to each other. Any number can replace the "*2*" to add difficulty to the activity.
- Students play *Find the Number* on a hundred number chart located at the front of the room, with each of the numbers covered by a Post-it or a small tag. One child calls out a number, like *45*, and a volunteer tries to identify where it is on the chart. The indicated Post-it is then lifted to check the guess.

4. Develop a sense of the magnitudes of whole numbers, commonly used fractions, and decimals.

- Children are presented with four jars of jelly beans — one with 3 beans in it, one with 19 beans in it, one with 52 beans in it, and one with 156. The teacher then asks *Which of these jars do you think has about 50 beans in it?* The students discuss their reasons for believing as they do.
- Second graders are challenged to guess how many sheets of paper are in the ream of paper on the front table. After everyone has made a guess, one student counts out 25 sheets from the top of the pile and places them next to the rest of the pile. Everyone is offered a chance to change their estimates and to discuss the reason for their change. Then students agree on a way to verify their guesses before trying to guess how many such reams it would take to reach the ceiling!

- Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework*. They discuss how many dinosaurs of different types might fit into the classroom.
- Students fold paper circles into halves, fourths, and eighths and are asked questions like: *Which would you rather have, a half of a cherry pie or a fourth of the pie? How about three-eighths of a pizza or one-fourth?*
- Students read or listen to a piece of children's literature that has fractions as its theme, such as *Eating Fractions* by Bruce McMillan.

5. Understand the various uses of numbers including counting, measuring, labeling, and indicating location.

- A kindergarten teacher announces to her class: *Boys and Girls! Great News! The principal told me that our class has just won FIVE!* A discussion then ensues regarding the need for that number to exist in some context, to have some unit or label before it makes sense.
- Second graders are given a stack of old magazines. They cut out any information which uses numbers and sort them according to how they are used: as page numbers, as prices, as dates, as addresses, and so on.
- The class takes a walk around the school or neighborhood pointing out to each other the numbers they see, and discuss how they are used.

6. Count and perform simple computations with money.

- Students use play money to show different combinations of coins that can be used to "buy" an object. For example, an 11¢ pencil can be bought with 11 pennies, a dime and a penny, one nickel and six pennies, or two nickels and a penny.
- Students earn 2¢ each day for attendance and 1¢ for good behavior. They keep their play money in a bank, count it regularly and use it to buy objects from a treasure chest.
- Students play *Spend a Dollar*. They each start with $1 (either as a bill or in change) and then roll one or two dice to find out how much they "spend" on that turn. They trade coins as needed. The student who spends all of her money first wins.
- Students play a shopping board game. They each begin with a given amount of money in coins. They roll two dice to determine how far they move each turn. As they land on a space, they must buy whatever is shown. Some spaces may provide refunds. The winner is the first person to go around the board and still have money left.
- Students' abilities to recognize coins and find the value of a group of coins are assessed by having each student select three objects to "buy," identify and name the coins needed to purchase each object, and find the total amount of money required to purchase all three.

7. **Use models to relate whole numbers, commonly used fractions, and decimals to each other, and to represent equivalent forms of the same number.**

    - When modeling 2-digit numbers with base-ten models such as popsicle sticks, base-ten blocks, or pennies and dimes, students are frequently asked to show all the ways they can make a given number. Children then begin to see that *3 tens and 7 ones, 2 tens and 17 ones, 1 ten and 27 ones,* and *37 ones* all represent the same number *37*.
    - Students each develop questions whose answers are all equivalent to some target number. For example, if the target is *8*, students may ask the following questions: *What is 4+4? What is 9-1? What is 8+0? How many hands do four children have? How many days is one more than a week?* or *How much is a nickel and three pennies?*
    - Students use geoboards, pattern blocks, Cuisenaire Rods, paper folding, and tangrams to explore simple common fractions like halves, thirds, and fourths. For instance, they may be challenged to model 1/2 with all of the different models.

8. **Compare and order whole numbers, commonly used fractions, and decimals.**

    - Young students use dot pattern cards or dominoes to practice *more, less,* and *same.* For example, given a card with 6 dots on it, students use counters to make a set that is more, another that is less, and one that is the same. They can then label the sets with cards that show the appropriate words. With dominoes, students work in pairs to compare the dots on the two halves and state which is more and by how much.
    - Students play the old favorite card game *war* with either dot cards or with a deck of regular playing cards minus the face cards. Every now and then, the rule changes so that the student with the card that is *less* wins the play.
    - Students play *Guess the Point*. A long number line with endpoints of 20 and 75, for example, is drawn on the board where all of the intermediary points are labeled above the line. The labels are then covered by a long piece of paper that can be lifted to reveal them. A student places a finger somewhere on the line and others must estimate the numerical label of the point chosen. The paper is then lifted to check the accuracy of their responses.

9. **Explore real-life settings which give rise to negative numbers.**

    - Primary classrooms are equipped with Celsius thermometers, in addition to Fahrenheit ones, so that "below zero" outdoor temperatures can be recorded. Temperature reports, possibly in both scales, become a part of the everyday calendar routine.

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National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA, 1989.

Software

Balancing Bear. Sunburst Communication.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

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Standard 7 — Geometry and Spatial Sense — Grades K-2

Overview

Students can develop strong spatial sense from consistent experiences in classroom activities that use a variety of manipulatives and technology. The key components of spatial sense, as identified in the K-12 Overview, are spatial relationships, properties of geometric figures, geometric transformations, coordinate geometry, geometry of measurement, geometric modeling, and reasoning.

In kindergarten through second grade, the emphasis is on qualitative, not quantitative, properties of geometric objects. Students are at the visualization level of geometric thinking, where they perceive figures as "wholes". They recognize squares and rectangles, but perhaps not that squares are a special case of rectangles. To enrich and develop their geometric thinking, children at these grade levels need to explore geometry using a variety of physical objects, drawings, and computer tools. They work with solids, pattern blocks, templates, geoboards, and computer drawing tools to develop their understanding of geometric concepts and their spatial sense. They construct models and drawings to experiment with orientation, position, and scale, and to develop visualization skills. Students begin to develop a geometric vocabulary. A sample unit on geometry for the second-grade level can be found in Chapter 17 of this *Framework*.

In their study of **spatial relationships**, students focus on developing their understanding of objects in space. They discuss and describe the relative positions of objects using phrases like "in front of" and "on top of." They describe and draw three-dimensional objects in different relative locations. They compare and contrast shapes, describing the shapes of the faces and bases of three-dimensional figures. They discuss symmetry and look for examples of symmetry in their environment. They look for shapes that are the same size and shape (congruent) or the same shape but different sizes (similar). They use mirrors to explore symmetry.

In beginning their study of **properties of geometric figures**, students look for shapes in the environment, make models from sticks and clay or paper and glue, and draw shapes. They sort objects according to shape. They recognize, classify, sort, describe, and compare geometric shapes such as the sphere, cylinder, cone, rectangular solid, cube, square, circle, triangle, rectangle, hexagon, trapezoid, and rhombus. They describe the angle at which two edges meet in different polygons as being smaller than a right angle, a right angle, or larger than a right angle. They discuss points, lines, line segments, intersecting and non-intersecting lines, and midpoints of lines.

Students begin looking at **geometric transformations** by using concrete materials such as paper dolls to model slides (translations), flips (reflections), and turns (rotations). Students put shapes together to make new shapes and take shapes apart to form simpler shapes. Students work on spatial puzzles, often involving pattern blocks or tangrams. They look for plane shapes in complex drawings and explore tilings. They divide figures into equal fractional parts, for example, by folding along one, two, or three lines.

Coordinate geometry in grades K-2 involves describing the motion of an object. Students make maps of real, imaginary, or storybook journeys. They describe the location of an object on a grid or a point in a

plane using numbers or letters. They give instructions to an imaginary "turtle" to crawl around the outline of a figure.

Students in these grades also begin to explore the **geometry of measurement**. In kindergarten, students discuss and describe quantitative properties of objects using phrases like "bigger" or "longer." They order objects by length or weight. In first and second grade, they quantify properties of objects by counting and measuring. They determine the areas of figures by cutting them out of grid paper and counting the squares. They measure the perimeter of a polygon by adding the lengths of all of the sides.

Students begin to explore **geometric modeling** by constructing shapes from a variety of materials, including toothpicks and clay, paper and glue, or commercial materials. They use templates to draw designs, and record what they have constructed out of pattern blocks and tangrams. They fold, draw, and color shapes. They copy geoboard figures, and construct them both from memory and by following oral or written instructions. They may also use geometric models, such as the number line, for skip counting or repeated addition.

Geometry provides a rich context in which to begin to develop students' **reasoning** skills. Students apply thinking skills in geometric tasks from identifying shapes to discovering properties of shapes, creating geometric patterns, and solving geometric puzzles and problems in a variety of ways. They create, describe, and extend geometric patterns. They use attribute blocks to focus on the similarities and differences of objects.

Geometry provides a unique opportunity to focus on the First Four Standards, especially Standard 2 which stresses the importance of making connections to other mathematical topics. For example, students often use their understanding of familiar shapes to help build an understanding of fractions. Teachers in grades K-2 need to plan classroom activities that involve several mathematical processes and relate geometry to other topics in mathematics. Geometry should not be taught in isolation; it should be a natural and integrated part of the entire curriculum.

Standard 7 — Geometry and Spatial Sense — K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Explore spatial relationships such as the direction, orientation, and perspectives of objects in space, their relative shapes and sizes, and the relations between objects and their shadows or projections.**
    - Blindfolded students are given real objects to touch and then, with the blindfolds removed, select the objects from a collection of visible objects.
    - Students work through the *Will a Dinosaurs Fit?* lesson that is described in the First Four Standards of this *Framework*. They discuss the size of the different dinosaurs and arrange them from smallest to largest.
    - Students predict what shape will result when a small piece is cut out of a folded piece of paper in different ways (along a diagonal, across a fold or a corner, or in the center) and the paper is then unfolded.
    - Students compare the sequence of objects seen from different points of view. For example, from the classroom window, the swings are to the left of the monkeybars, but the relationship is reversed if the objects are viewed from the blacktop facing the classroom.
    - Students predict and draw what the shadow of an object placed between a light and a screen will look like.

2. **Explore relationships among shapes, such as congruence, symmetry, similarity, and self-similarity.**
    - Students look for examples of congruent figures (same size and shape) in the environment.
    - Students explore symmetry by using mirrors with pattern blocks or by folding paper or by making inkblot designs. Students find the lines of symmetry in the letters of the alphabet and in numerals. They fold paper and cut out symmetric designs. They identify the symmetry in wallpaper or giftwrap designs.
    - Students use different size dolls and action figures as an introduction to the concept of similarity (same shape, different size).

3. **Explore properties of three- and two-dimensional shapes using concrete objects, drawings, and computer graphics.**
    - Students predict what shape they will see when they make various impressions of 3-dimensional objects in sand. For example, the top of a cylinder forms a circle, its side forms a rectangle.

- Students outline a triangle, a square, and a circle on the floor with string or tape. Then they walk around each figure, chanting a rhyme, such as "Triangle, triangle, triangle, 1, 2,3, I can walk around you as easy as can be," and counting the sides as they walk.
- Students work through the *Shapetown* lesson that is described in the First Four Standards of this *Framework*. They explore properties of two-dimensional shapes by applying the fundamental concepts of Venn diagrams.
- Some students use *Muppet™ Math* to work with Kermit's geometric paintings, while others use *Shape Up!* to compare everyday objects to geometric shapes.

4. Use properties of three- and two-dimensional shapes to identify, classify, and describe shapes.

- Students make shapes with their fingers and arms.
- Students listen to and look at the book *The Shapes Game* by Paul Rogers. Each page shows a different shape and many of the things in the world that have that shape. As each page is read, the children find other objects in the room that have the same shape.
- Students listen to and draw illustrations for the story *The Greedy Triangle* by Marilyn Burns.
- A good open-ended assessment for this critical indicator is to ask students to sort a collection of shapes into groups, explaining their reasoning. Some groups they might consider include "all right angles" or "four-siders." The teacher should encourage the students to invent appropriate group names and to use informal language to describe the properties, and should record the students' responses to look for progress over time.
- A more traditional, but still useful, assessment strategy is to ask students to sort pictures cut from magazines according to shape. This more focused task will generate information about the students' ability to recognize and differentiate among shapes.
- Students make class books shaped like a triangle, a rectangle, a square, and a circle. They fill each book with pictures of objects that have the shape of the book.
- Students turn a geometric shape into a picture. For example, a triangle might become a tower, a clown face, or the roof of a house.

5. Investigate and predict the results of combining, subdividing, and changing shapes.

- Students use tangram pieces to construct triangles, rectangles, squares, and other shapes.
- Students investigate which pattern block shapes can be formed from the equilateral triangles, recording their results in pictures and on a chart.
- Students work in groups to decide how to divide a rectangular candy bar among four people. The students then compare the various ways that each group solved the problem.

6. Use tessellations to explore properties of geometric shapes and their relationships to the concepts of area and perimeter.

- Students use Unifix cubes or pattern blocks to create colorful designs. They then discuss how many blocks they used (area) and the distance around their design (perimeter). They also discuss why these polygon shapes fit together like a puzzle.

- Students use different shapes to make quilt patterns.
- During free play time, students use pattern blocks to make different space-filling designs. They record any patterns that they especially like, using templates or drawing around the blocks.

7. Explore geometric transformations such as rotations (turns), reflections (flips), and translations (slides).

- Students look at the world around them for examples of changes in position that do not change size or shape. For example, a child going down a slide illustrates a slide, a merry-go-round or hands on a clock illustrate a turn, and a mirror illustrates a flip.
- Students look through and discuss the no-text book *Changes, Changes* by Pat Hutchins. In it, a man and a woman use the same set of building blocks to transform a house into a fire engine, then a boat, a truck, and back to a house. The students tell the story and then draw pictures to show how the blocks changed from one object to another.
- Students investigate the shapes that they can see when they place a mirror on a square pattern block.

8. Develop the concepts of coordinates and paths, using maps, tables, and grids.

- Students use maps of their community to find various ways to get from school to the park. They use letters and numbers to describe the location of the school and that of the park.
- Students create a map based on the familiar story of *The Little Gingerbread Man*, showing where each of the people in the story lives.
- Students describe how to get from one point in the school to another and try to follow each others' directions.

9. Understand the variety of ways in which geometric shapes and objects can be measured.

- In connection with a unit on dinosaurs in science, students discuss the different ways in which the size of dinosaurs can be described. They decide to measure the size of a dinosaur's footprint in two ways: by using string to measure the distance around it and by using base ten blocks to measure the space inside it.
- Pairs of students investigate the many different designs that they can make using unit squares and 1/2-unit right triangles. They record their results on dot paper.

10. Investigate the occurrence of geometry in nature, art, and other areas.

- Students take a "geometry walk" through their school or their neighborhood, looking for examples of specific shapes and concepts.
- Students create geometric patterns using potato prints.
- Students decorate their classroom for the winter holidays using geometric shapes.
- Students examine and discuss geometric patterns found in works of art.

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Overview

The wide availability of computing and calculating technology has given us the opportunity to significantly reconceive the role of computation and numerical operations in our elementary mathematics programs, but, in kindergarten through second grade, the effects will not be as evident as they will be in all of the other grade ranges. This is because the numerical operations content taught in these grades is so basic, so fundamental, and so critical to further progress in mathematics that much of it will remain the same. The approach to teaching that content, however, must still be changed to help achieve the goals expressed in the *New Jersey Mathematics Standards*.

Learning the **meanings of addition and subtraction,** gaining facility with **basic facts**, and mastering some **computational procedures for multi-digit addition and subtraction** are still the topics on which most of the instructional time in this area will be spent. There will be an increased conceptual and developmental focus to these aspects of the curriculum, though, away from a traditional drill-and-practice approach, as described in the K-12 Overview; nevertheless, students will be expected to be able to respond quickly and easily when asked to recall basic facts.

By the time they enter school, most young children can use counters to act out a mathematical story problem involving addition or subtraction and find a solution which makes sense. Their experiences in school need to build upon that ability and deepen the children's understanding of the **meanings of the operations**. School experiences also need to strengthen the children's sense that modeling such situations as a way to understand them is the right thing to do. It is important that they be exposed to a variety of different situations involving addition and subtraction. Researchers have separated problems into categories based on the kind of relationships involved (Van de Walle, 1990, pp. 75-6); students should be familiar with problems in all of the following categories:

Join problems

- Mary has 8 cookies. Joe gives her 2 more. *How many cookies does Mary have in all?*
- Mary has some cookies. Joe gives her 2 more. Now she has 8. *How many cookies did Mary have to begin with?* (Missing addend)
- Mary has 8 cookies. Joe gives her some more. Now Mary has 10. *How many cookies did Joe give Mary?* (Missing addend)

Separate problems

- Mary has 8 cookies. She eats 2. *How many are left?* (Take away)
- Mary has some cookies. She eats 2. She has 6 left. *How many cookies did Mary have to begin with?*
- Mary has 8 cookies. She eats some. She has 6 left. *How many cookies did Mary eat?* (Missing addend)

Part-part-whole problems

- Mary has 2 nickels and 3 pennies. *How many coins does she have?*
- Mary has 8 coins. Three are pennies, the rest nickels. *How many nickels does Mary have?*

Compare problems

- Mary has 6 books. Joe has 4. *How many more books does Mary have than Joe?*

- Mary has 2 more books than Joe. Mary has 6 books. *How many books does Joe have?*
- Joe has 2 fewer books than Mary. He has 4 books. *How many books does Mary have?*

Basic facts in addition and subtraction continue to be very important. Students should be able to quickly and easily recall one-digit sums and differences. The most effective way to accomplish this has been shown to be the focused and explicit use of basic fact strategies—conceptual techniques that make use of the child's understanding of number parts and relationships to help recover the appropriate sum or difference. By the end of second grade, students should not only be able to use *counting on*, *counting back*, *make ten*, and *doubles* and *near doubles* strategies, but also explain why these strategies work by modeling them with counters. Building on their facility with learning doubles like *7 + 7 = 14*, children recast *7 + 8* as *7 + 7 + 1*, which they then recognize as 15 (*near doubles*). *Make ten* involves realizing that in adding *8 + 5*, you need two to make ten, and recasting the sum as *8 + 2 + 3* which is *10 + 3* or *13*. *Counting on* involves starting with the large number and counting on the smaller number so that adding *9 + 3*, for example, would involve counting on *10*, *11*, and then *12*. *Counting back* is used for subtraction, so that finding *12 – 4*, the child might count *11, 10, 9*, and then *8*.

Students must still be able to perform **multi-digit addition and subtraction** with paper and pencil, but the widespread availability of calculators has made the particular procedure used to perform the calculations less important. It need no longer be the single fastest, most efficient algorithm chosen without respect to the degree to which children understand it. Rather, the teaching of multi-digit computation should take on more of a problem solving approach, a more conceptual, developmental approach. Students should first use the models of multi-digit number that they are most comfortable with (base ten blocks, popsicle sticks, bean sticks) to explore the new class of problems. Students who have never formally done two-digit addition might be asked to use their materials to help figure out how many second graders there are in all in the two second grade classes in the school. Other similar real-world problems should follow, some involving regrouping and others not. After initial exploration, students share with each other all of the strategies they've developed, the best ways they've found for working with the tens and ones in the problems, and their own approaches (and names!) for regrouping. Most students can, with direction, take the results of those discussions and create their own paper-and-pencil procedures for addition and subtraction. The discussions can, of course, include the traditional approaches, but these ought not to be seen as *the only right way* to do these operations.

Kindergarten through second grade teachers are also responsible for setting up an atmosphere where **estimation** and **mental math** are seen as reasonable ways to do mathematics. Of course students at these grade levels do almost exclusively mental math until they reach multi-digit operations, but estimation should also comprise a good part of the activity. Students regularly involved in real-world problem solving should begin to develop a sense of when estimation is appropriate and when an exact answer is necessary.

Technology should also be an important part of the environment in primary classrooms. Calculators provide a valuable teaching tool when used to do student-programmed skip counting, to offer estimation and mental math practice with *target games*, and to explore operations and number types that the students have not formally encountered yet. They should also be used routinely to perform computation in problem solving situations that the students may not be able to perform otherwise. This use prevents the need to artificially contrive the numbers in real-world problems so that their answers are numbers with which the students are already comfortable.

The topics that should comprise the numerical operations focus of the kindergarten through second grade

mathematics program are:

addition and subtraction basic facts
multi-digit addition and subtraction

Standard 8 — Numerical Operations — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Develop meaning for the four basic arithmetic operations by modeling and discussing a variety of problems.**
    - Students use unifix cube towers of two colors to show all the ways to make *"7"* (for example: *3+4, 2+5, 0+7*, and so on). This activity focuses more on developing a sense of *"sevenness"* than on addition concepts, but a good sense of each individual number makes the standard operations much easier to understand.
    - Kindergartners and first graders use workmats depicting various settings in which activity takes place to make up and act out story problems. On a mat showing a vacant playground, for instance, students place counters to show 3 kids on the swings and 2 more in the sandbox. *How many kids are there in all? How many more are on the swings than in the sandbox? What are all of the possibilities for how many are boys and how many are girls?*
    - Students work through the *Sharing a Snack* lesson that is described in the Introduction to this *Framework*. It challenges students to find a way to share a large number of cookies fairly among the members of the class, promoting discussion of early division, fraction, and probability ideas.
    - Students learn about addition as they read *Too Many Eggs* by M. Christina Butler. They place eggs in different bowls as they read and then make up addition number sentences to find out how many eggs were used in all.
    - Kindergarteners count animals and learn about addition as they read *Adding Animals* by Colin Hawkins. This book uses addends from one through four and shows the number sentences that go along with the story.
    - Students are introduced to the take-away meaning for subtraction by reading *Take Away Monsters* by Colin Hawkins. Students see the partial number sentence (e.g., *5 – 1 =*), count to find the answer, and then pull the tab to see the result.
    - Students explore subtraction involving missing addend situations as they read *The Great Take-Away* by Louise Mathews. This book tells the story of one lazy hog who decides to make easy money by robbing the other pigs in town. The answers to five subtraction mysteries are revealed when the thief is captured.
    - Students make booklets containing original word problems that illustrate different addition or subtraction situations. These may be included in a portfolio or evaluated independently.
2. **Develop proficiency with and memorize basic number facts using a variety of fact strategies**

(such as "counting on" and "doubles").

- Students play *one more than* dominoes by changing the regular rules so that a domino can be placed next to another only if it has dots showing *one more than* the other. Dominoes of any number can be played next to others that show *6* (or *9* in a set of double nines). *One less than* dominoes is also popular.
- Students work through the *Elevens Alive* lesson that is described in the Introduction to this *Framework*. It asks them to consider the parts of eleven and the natural, random, occurrence of different pairs of addends when tossing eleven two-colored counters.
- Second graders regularly use the *doubles* and *near doubles*, the *make ten*, and the *counting on* and *counting back* strategies for addition and subtraction. Practice sets of problems are structured so that use of all of these strategies is encouraged and the students are regularly asked to explain the procedures they are using.
- Students play games like *addition war* to practice their basic facts. Each of two children has half of a deck of playing cards with the face cards removed. They each turn up a card and the person who wins the trick is the first to say the sum (or difference) of the two numbers showing. Calculators may be used to check answers, if necessary.
- Students use the calculator to count *one more than* by pressing + *1* = = =. The display will increase by one every time the student presses the = key. Any number can replace the *1* key.
- Students use two dice to play board games (*Chutes and Ladders* or home-made games). These situations encourage rapid recall of addition facts in a natural way. In order to extend practice to larger numbers, students may use 10-sided dice.
- Students use computer games such as *Math Blaster Plus* or *Math Rabbit* to practice basic facts.

3. Construct, use, and explain procedures for performing whole number calculations in the various methods of computation.

- Second graders use popsicle sticks bundled as tens and ones to try to find a solution to the first two-digit addition problem they have formally seen: *Our class has 27 children and Mrs. Johnson's class has 26. How many cupcakes will we need for our joint party?* Solution strategies are shared and discussed with diversity and originality praised. Other problems, some requiring regrouping and others not, are similarly solved using the student-developed strategies.
- Students use calculators to help with the computation involved in a first-grade class project: to see how many books are read by the students in the class in one month. Every Monday morning, student reports contribute to a weekly total which is then added to the monthly total.
- Students look forward to the hundredth day of school, on which there will be a big celebration. On each day preceding it, the students use a variety of procedures to determine how many days are left before day 100.

- As part of their assessment, students explain how to find the answer to an addition or subtraction problem (such as *18 + 17*) using pictures and words.
- Students find the answer to an addition or subtraction problem in as many different ways as they can. For example, they might solve *28 + 35* in the following ways:

  8 + 5 = 13 and 20 + 30 = 50, so 13 + 50 = 63

  28 + 30 = 58. Two more is 60, and 3 more is 63

  25 + 35 = 60 and 3 more is 63.
- Students use estimation to find out whether a package of 40 balloons is enough for everyone in the class of 26 to have two balloons. They discuss the strategies they use to solve this problem and decide if they should buy more packages.

4. Use models to explore operations with fractions and decimals.

- Kindergartners explore part/whole relations with pattern blocks by seeing which shapes can be created using other blocks. You might ask: *Can you make a shape that is the same as the yellow hexagon with 2 blocks of some other color? with 3 blocks of some other color? with 6 blocks of some other color?* and so on.
- Students use paper folding to begin to identify and name common fractions. You might ask: *If you fold this rectangular piece of paper in half and then again and then again, how many equal parts are there when you open it up?* Similarly folded papers, each representing a different unit fraction, allow for early comparison activities.
- Second graders use fraction circles to model situations involving fractions of a pizza. For example: *A pizza is divided into six pieces. Mary eats two pieces. What fraction of the pizza did Mary eat? What fraction is left?*
- Students use manipulatives such as pattern blocks or Cuisenaire rods to model fractions. For example: *If the red rod is one whole, then what number is represented by the yellow rod?*

5. Use a variety of mental computation and estimation techniques.

- Students regularly practice a variety of oral counting skills, both forward and backward, by various steps. For instance, you might instruct your students to: *Count by ones — start at 1, at 6, at 12, from 16 to 23; Count by tens — start at 10, at 30, at 110, at 43, at 67, from 54 to 84*, and so on.
- Students estimate sums and differences both before doing either paper-and-pencil computation or calculator computation and after so doing to confirm the reasonableness of their answers.
- Students are given a set of index cards on each of which is printed a two-digit addition pair (*23+45, 54+76, 12+87*, and so on). As quickly as they can they sort the set into three piles: *more than 100, less than 100,* and *equal to 100.*
- Students play "*Target 50*" with their calculator. One student enters a two-digit number and the other must add a number that will get as close as possible to *50*.

6. **Select and use appropriate computational methods from mental math, estimation, paper-and-pencil, and calculator methods, and check the reasonableness of results.**

   - The daily *calendar routine* provides the students with many opportunities for computation. Questions like these arise almost every day: *There are 27 children in our class. Twenty-four are here today. How many are absent? Fourteen are buying lunch; how many brought their lunch?* or *It's now 9:12. How long until we go to gym at 10:30?* The students are encouraged to choose a computation method with which they feel comfortable; they are frequently asked why they chose their method and whether it was important to get an exact answer. Different solutions are acknowledged and praised.
   - Students regularly have *human vs. calculator races*. Given a list of addition and subtraction basic facts, one student uses mental math strategies and another uses a calculator. They quickly come to realize that the human has the advantage.
   - Students regularly answer multiple choice questions like these with their best guesses of the most reasonable answer: *A regular school bus can hold: 20 people, 60 people, 120 people? The classroom is: 5 feet high, 7 feet high, 10 feet high?*
   - As part of an assessment, students tell how they would solve a particular problem and why. They might circle a picture of a calculator, a head (for mental math), or paper-and-pencil for each problem.

7. **Understand and use relationships among operations and properties of operations.**

   - Students explore three-addend problems like *4 + 5 + 6 =*. First they check to see if adding the numbers in different orders produces different results and, later, they look for pairs of compatible addends (like *4* and *6*) to make the addition easier.
   - Students make up humorous stories about adding and subtracting zero. *I had 27 cookies. My mean brother took away zero. How many did I have left?*
   - Second graders, exploring multiplication arrays, make a *4* x *5* array of counters on a piece of construction paper and label it: *4 rows, 5 in each row = 20*. Then they rotate the array *90°* and label the new array, *5 rows, 4 in each row = 20*. Discussions follow which lead to intuitive understandings of commutativity.

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On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

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Standard 9 — Measurement — Grades K-2

Overview

Students can develop a strong understanding of measurement and measurement systems from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding, as identified in the K-12 Overview, are: **the concept of a measurement unit; standard measurement units; connections to other mathematical areas and to other disciplines; indirect measurement**; and, for older students, **measurement error** and **degree of precision**.

Students in the early grades encounter measurement in many situations, from their daily work with the calendar, to situations in stories that they are reading, to describing how quickly they are growing. Hands-on science activities often require students to measure objects or compare them directly. Daily calendar activities frequently offer work with temperature, time, and money, in addition to number. Thus, many opportunities for **connections** present themselves in a natural way.

The study of measurement also encourages students to develop their number sense and to practice their counting skills. By using measures, students can recognize that numbers are often used to describe and compare properties of physical objects. Students in the early grades should make estimates not only of discrete objects like marbles or seeds but also of continuous properties like the length of a jumprope or the number of children's feet which might fit in a dinosaur's footprint.

Students need to focus on identifying the property that they wish to measure. They should understand what is meant by the length of an object or its weight or its capacity. Concrete experiences in describing the properties of objects, in sorting objects, and in comparing and contrasting objects provide them with opportunities to develop these concepts.

Students begin by making direct comparisons. *Which string is longer? Which child is taller? Which rock is heavier? Which glass holds more?* Making comparisons will help children better to understand the properties which they are discussing. They also begin to make some **indirect measurements.** For example, in order to compare the height of the blackboard with the height of a window, they might measure both objects using links and then compare the number of links used for each. In this way, they start to see a need for a **measurement unit,** a unit that they can use over and over to compare to a variety of objects.

In grades K-2, students should use a variety of non-standard units to measure objects. *How many links long is a desk? How many erasers high are you? How many pennies balance a Unifix cube?* In each case, students should first be asked to make an estimate and then proceed to actually measure the object. Students should also use different units to measure the same object. They should begin to understand that when the size of a measuring unit increases, the number of units needed to measure the object decreases.

In these grades, students also begin to use **standard measurement units** and standard measurement devices such as rulers and scales. It is important that the students see the use of the standard devices as simply an extension of their earlier activities. For example, the use of an inch ruler is just a more efficient procedure than lining up a series of cubic inch blocks. Students should explore length using inches, feet, centimeters and meters; liquid capacity using quarts, pints, cups, and liters; mass/weight using pounds, ounces, grams,

and kilograms; time using days, weeks, months, years, seconds, minutes, and hours; and temperature using degrees Fahrenheit and Celsius.

Whether making direct comparisons, using non-standard units, or using standard measurement units, students in the early grades should always estimate a measure first and then perform the measurement. In this way, their estimation and number sense skills will be reinforced.

Standard 9 — Measurement — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Use and describe measures of length, distance, capacity, weight, area, volume, time, and temperature.**
    - Students find out how many cubes long their hand is. The class can then generate a graph showing the results.
    - Using a large map of the school community, students estimate and then use paper clips or links to measure who lives farthest from school. This type of activity might be related to a specific story that was used in the Social Studies unit on community.
    - Students name objects big enough to hold a football or too small to hold a soccer ball.
    - Students lay out a model zoo with several toy animals, using boxes of different sizes for their cages or yards. They also cut doors of appropriate sizes in the boxes for the animals.
    - Students listen to and look at the book *Let's Find Out about What's Light and What's Heavy* by Martha and Charles Shapp. The simple text and humorous illustrations lead to the conclusion that weighing things using a standard unit of weight helps answer the question in the title.
    - Students name objects they can lift and ones that they cannot lift.
    - Students estimate and then use balances to find out how many pennies balance a small familiar object.
    - Students cut strips of paper to fit around a pumpkin or to make Santa's belt.
    - Students fill a large bottle with water using first a 4 ounce cup and then an 8 ounce cup. They then compare the results.
    - Students make their own measuring jug using a large plastic jar. They pour in one cupful of water and mark the water level on the jar with a marker; they repeat this procedure with one cupful after another until no further cupfuls will fit inside the jar.
    - Students read *The Little Gingerbread Man* and make a gingerbread village. In doing so, they measure lengths and capacities.
    - Students make their own paper clip ruler. First they make a paper clip chain and then paste it down on a long cardboard strip. They draw a small vertical line where each paper clip ends.
    - Students estimate and measure the distance around an object using Unifix cubes or a paper clip chain.

- Students conduct experiments using timers: *How many times can you bounce a ball before all the water runs out of the can? How many times can you clap your hands before the sand runs out of the timer? How many times can you blink your eyes before the second hand goes all the way around the clock?*
- Students read or listen to *The Very Hungry Caterpillar* by Eric Carle which shows the time of day at which various activities occur.
- Students make a book describing their day at school. On each page, they stamp a clock face and write underneath a time that the teacher has written on the board. They then draw the hands on the face to show the time. When the actual time of day on the classroom clock matches a time in their book, students draw a picture of what they are doing next to the correct clock face.
- Students line up Cuisenaire Rods in different combinations to measure the width of a sheet of paper.

2. Compare and order objects according to some measurable attribute.

- Kindergarten students listen to and look at the book *Big Friend, Little Friend* by Eloise Greenfield. In it a young boy and his two friends explore situations that clearly demonstrate what it means to be big and what it means to be little. As a nice follow-up assessment activity to this and other manipulative activities, the teacher has the students draw pictures which illustrate "big and little."
- Students compare the lengths of pencils to find out which is longest. They arrange a set of pencils in order from longest to shortest.
- Students use water, rice, or sand to fill different objects, pouring from one object into another to find out which object holds more. They explain how the shape of each object plays a role in the amount it holds.
- Students line up in order, from tallest to shortest.
- Children make stick drawings of a family: father, mother, school-aged child, and baby. They discuss which stick drawings are taller or shorter than others, and relate these to the relative size of the individuals in the family.
- Students collect a large variety of cardboard boxes and arrange them in order from smallest to largest.
- Each group of students is given a cup and several containers of different sizes, plain white paper, some uncooked rice, and 1 inch graph paper. They find out how many cups of rice will fit in each container and show the number of cups as a bar on the graph paper under a picture of the container. After completing the graph, they arrange the containers in order from largest to smallest.
- Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework.* They arrange the dinosaurs in order from tallest to smallest according to their height, and from longest to shortest according to their length.

3. Recognize the need for a uniform unit of measure.

- Students measure the width of their desks by counting how many widths of their hands it

would take to go from one end of the desk to the other. They compare their results and discuss what would happen to the number of hands if the teacher's hand were used instead.

- Students read and discuss *How Big Is a Foot?* by Rolf Myllar. The king wishes to give the queen a special bed for her birthday and measures the size using his own foot. He gives the measurements to the carpenter, who gives them to the little apprentice. The bed that he makes is too small, but the apprentice solves the problem and everyone lives happily ever after. The students use their own feet to measure the width or length of the hallway and compare their results. Finally, they measure the hallway using meter sticks.
- As an assessment of the students' understanding of units, the teacher has the students measure the length of their math book using paper clips, unifix cubes, and yellow Cuisenaire Rods. They write about their results and explain why they are different.

4. **Develop and use personal referents for standard units of measure (such as the width of a finger to approximate a centimeter).**
    - Students identify parts of their body that are the same length as the unit cube from a base tens block set (1 centimeter).
    - Students make a list of foods or drinks that come in quarts and others that come in liters.
    - Students find out that ten pennies weigh about an ounce.
    - Students find that first-graders are a little taller or a little shorter than a meter.

5. **Select and use appropriate standard and non-standard units of measurement to solve real-life problems.**
    - Students decide whether they should use paper clips or pennies to measure the weight of a pencil.
    - Students discuss whether they should use links or meter sticks to measure the length of the gym.
    - Students write about how they might measure the distance from the cafeteria to their classroom.

6. **Understand and incorporate estimation and repeated measures in measurement activities.**
    - Students estimate how many of their shoes will fit in a giant's footprint (left conveniently on the classroom blackboard!) and write their estimates. They trace around their shoes and cut out the tracings. After the teacher has pasted a few shoes onto the giant's footprint, the students revise their estimate. They then check the accuracy of their estimates by pasting as many shoes as will fit into the footprint.
    - Students estimate the weight of various objects in beans and then use a balance scale to check the accuracy of their measurements.

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One-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

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Standard 10 — Estimation — Grades K-2

Overview

As indicated in the K-12 Overview, students' ability to use estimation appropriately in their daily lives develops as they focus on the **reasonableness of answers**, explore and construct **estimation strategies**, and **estimate measurements**, **quantities**, **and the results of computation**.

One of the estimation emphases for very young children is the development of the idea that guessing is an important and exciting part of mathematics. The teacher must employ sound management practices which ensure that everyone's guess is important and which encourage student risk-taking and sharing of ideas about how their guesses were determined. When first asked to guess an answer, many students will give nonsense responses until they establish appropriate experiences, build their sense of numbers, and develop informal strategies for creating a guess. Children begin to make reasonable estimates when the situations involved are relevant to their immediate world. Building on comparisons of common objects and using personal items to build a sense of lengths, weights, or quantities helps children gain confidence in their guessing. As children communicate with each other about how guesses are formulated they begin to develop informal **strategies for estimation**.

Estimation with computation is as important at these early grade levels as it is at all the other grade levels. Estimation of sums and differences should be a part of the computational process from the very first activity with any sort of computation. Children should regularly be asked *About how many do you think there will be in all?* or *About what do you think the difference is?* or *About how many do you think will be left?* in the standard addition and subtraction settings. These questions are appropriate whether or not exact computations will be done. Children should understand that, sometimes, the estimate will be an accurate enough number to serve as an answer. At other times, an exact computation will need to be done, either mentally, with paper-and-pencil, or with a calculator to arrive at a more precise answer. The particular procedure to be used is dependent on the setting and the problem.

One of the most useful computational **estimation strategies** at these grade levels also reinforces an important place value idea. Students should understand that in two-digit numbers the tens digit is much more meaningful than the ones digit in contributing to the overall value of the number. A reasonable approximation, then, of a two-digit sum or difference can always be made by considering only the tens digits and ignoring the ones. This strategy is referred to as *front end estimation* and is used with larger numbers as well, although then the first two digits may be used. It is the main estimation strategy that many adults use.

Standard 10 — Estimation — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and in grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set.**
    - Students place various amounts of counters or other small objects in individual plastic bags. Working in groups of four students, the children choose one bag to be the reference set and judge whether each of the other bags has more than, less than, or the same as the reference set. Initially, they should try to make the judgments without counting. The teacher observes the groups as they work, making notes about the students' progress.
    - Young children benefit from frequently comparing sets of objects to some given number. For example, given sets of colored chips arranged on a table, they should name which sets have more than five and which have less than five.
    - Students play the card game *War* with a set of cards without numerals (i.e., cards which only show sets of hearts, clubs, diamonds, or spades). Students will easily distinguish between the 7's and the 3's, but will be reluctant to make judgments about closer numbers like the 4's and 5's without counting. As they play more often, however, their ability to distinguish will visibly improve. They may also begin to notice patterns involving even and odd numbers on their own.
    - Students learn to recognize certain arrangements of dots or stars as representing certain numbers. Using flashcards, they estimate the number of dots or stars, and then count to check their estimates.
    - After reading *Ten Black Dots* by Donald Crews, students make up their own uses for 1-10 black dots. They use adhesive dots to create their own books that include uses for each of the numbers from 1 through 10. They then estimate and count the total number of dots they actually use for their book. (The total might surprise them: 55.)
    - As an assessment of students' ability to judge without counting, the teacher puts some counters (more than five) on the overhead projector, turns it on for a few seconds, and then asks the students to write whether the number of counters shown is closer to 10 or to 20.

2. **Use personal referents, such as the width of a finger as one centimeter, for estimations with measurement.**
    - Students estimate lengths of pieces of spaghetti, yarn, paper, pencils, paper clips, etc., using suggested non-standard personal units such as width of thumb, length of a foot, and so on. They note that different students get different "right" answers.
    - As standard units like foot and centimeter are introduced, students are challenged to find

some part of their body or some personal action that is about that size at this point in their growth. For instance, they may decide that the width of their little finger is almost exactly one centimeter or the length of one baby step is one foot.

- Students use their self-discovered personal body referents to estimate the measures of various classroom objects like the length of the blackboard or the width of a piece of paper. They compare their answers, noting that when larger units are used, the estimated answer is a smaller number; and when smaller units are used, the estimated answer is a larger number.

3. Visually estimate length, area, volume, or angle measure.

- Students look at a quantity of sand, salt, flour, water, macaroni, corn, or popcorn and estimate how many times it could fill up a specified container.
- Students estimate how many pieces of notebook paper it would take to cover a given area such as the blackboard, or a portion of the classroom floor.
- Students regularly estimate lengths using a variety of non-standard units such as *my feet, Unifix cubes, paper clips,* and *orange Cuisenaire Rods.* They then measure to verify or revise their estimates.
- Students begin to develop an understanding of angle measure by making right-hand (or left-hand) turns repeatedly to turn completely around. They also compare angles to right-angle "corners," and decide whether an angle is more than or less than a "corner."
- Students note that there are 12 numbers on a clock face and discuss how far each hand moves in an hour. They note that each hand moves in a circle, but that the hour hand moves much more slowly.
- Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework.* They determine the size of the room, hear their classmates' presentations about the dinosaurs, and then as a whole class activity estimate which dinosaurs, and how many of them, might fit into the room.

4. Explore, construct, and use a variety of estimation strategies.

- Students are asked if a sixty-seat bus will be adequate to take the two first grade classes on their field trip. After it is known that there are 23 children in one class and 27 in the other, individuals volunteer their answers and give a rationale to support their thinking; *front end estimation* should lead to the conclusion that the total number of students is between 40 and 60. A discussion might be directed to the question of whether an exact answer to the computation was needed for the problem.
- Students are shown a glass jar filled with about eighty marbles and asked to estimate the number in the jar. In small groups, they discuss various approaches to the problem and strategies they can use. Each group shares one strategy with the class, and the estimate that resulted. The teacher makes notes about students' work throughout the activity.
- Second grade students can be challenged to estimate the total number of students in the school. They will need to talk informally about the average number of students in each class, the number of classes in a grade level, and the number of grade levels in the school. They might then use calculators to get an answer, but the result, even though the exact answer to a

computation, is still an estimate to the original problem. They discuss why that is so.

- Primary-grade students explore the meanings of comparison words by listening to *How Many is Many?* by Margaret Tuten. They compare *big* and *small, long* and *short,* a *lot* and a *few*. They list how many pieces of candy would be a few and how many pieces would be many, eventually reaching general agreement, perhaps on 5 as a few. Then they consider whether 5 teaspoons of medicine would be a few.

5. Recognize when estimation is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.

- Given a pair of real-life situations, students determine which situation in the pair is the one for which estimation is a good approach and which is the one that probably requires an exact answer. One such pair, for example, might be: *sharing a bag of peanuts among 3 friends* and *paying for 3 tickets at the movie theater*.
- Given a set of cartoons with home-made mathematical captions, first graders decide which of the cartoon characters arrived at exact answers and which got estimates. Two of the cartoons might show an adult and a child looking at a jar of jellybeans and the captions might read: *Susie guessed that there were 18 jellybeans left in the jar* and *Susie's mom counted the 14 jellybeans left in the jar.*
- Students read or listen to newspaper headlines and discuss which involve exact numbers and which might be estimates.

6. Determine the reasonableness of an answer by estimating the result of operations.

- Students are regularly asked if their answer makes sense in the context of the problem they were solving. They respond with full sentences explaining what they were asked to find and why the numerical answer they found fits the context reasonably, that is, why it *could be* the answer. For example, first graders might be asked to decide if their answer to the following problem makes sense: *Mary made 27 cookies and Jose made 15. How many cookies did they make in all?* Some responses might indicate that the answer should be more than *20 + 10 = 30* and less than *30 + 20 = 50*. Other students might say that they know that *25* and *15* is *40*, so the answer should be a little more.
- Students estimate *reasonable* numbers of times that particular physical feats can be performed in one minute. For example: *How many times can you bounce a basketball in a minute? How many times can you hop on one foot in a minute? How many times can you say the alphabet in a minute?* and so on. Other students judge whether the estimates are reasonable or unreasonable and then the tasks are performed and the actual counts made.
- Second-grade students are given a set of thirty cards with two-digit addition problems on them. In one minute, they must sort the cards into two piles: those problems whose answers are greater than 100 and those less than 100. The correct answers can be on the backs of the cards to allow self-checking after the task is completed.
- Second-grade students are given a page of addition or subtraction problems in a multiple choice format with 4 possible answers for each problem. Within some time period which is much too short for them to do the computations, students are asked to choose the most reasonable answer from each set of four.

7. **Apply estimation in working with quantities, measurement, time, computation, and problem solving.**

   - Students have small pieces of yarn of slightly different lengths ranging from 2 to 6 inches. Each student first estimates the number of his or her pieces it would take to match a much longer piece — about 30 inches long — and then actually counts how many. Then they use their individual pieces to measure other objects in the room. Each child is responsible for estimating the lengths in terms of his or her own yarn, but they can use evidence from other children's measuring to help make their own estimates.
   - Students regularly estimate in situations involving classroom routines. For example, at snack time, they may guess how many cups can be filled by each can of juice or how many crackers each student will get if all of the crackers in the box are given out.
   - Kindergartners always have fun deciding which color is best represented in a group of multi-colored objects. Good examples of such an activity would be choosing the color that shows up most often (or least often) in bags of M&M's, in handfuls of small squares of colored paper, or in a jar full of marbles. After everyone has committed to a guess, the children can sort the objects and count each color. They can then make bar graphs to show the distribution of the different colors.
   - Students use Tana Hoban's photographs in *Is It Larger? Is It Smaller?* as a starting point for investigating and comparing quantities and measures in their classroom. For example, on one page, three vases are shown filled with three different kinds of flowers. The reader must decide which objects to compare, such as the vases, before ordering them — from tallest to shortest and/or by volume.

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Tuten, Margaret. *How Many is Many?* Chicago: Children's Press, 1970.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 11 — Patterns, Relationships, and Functions — Grades K-2

Overview

The development of *pattern-based thinking*, using patterns to analyze and solve problems, is an extremely powerful tool for doing mathematics, and leads in later grades to an appreciation of how functions are used to describe relationships. The key components of *pattern-based thinking* at the early grade levels, as identified in the K-12 Overview, are **recognizing, constructing, and extending patterns, categorizing and classifying objects,** and **discovering rules.**

> "Looking for patterns trains the mind to search out and discover the similarities that bind seemingly unrelated information together in a whole. . . . A child who expects things to 'make sense' *looks* for the *sense* in things and from this sense develops understanding. A child who does not see patterns often does not *expect* things to make sense and sees all events as discrete, separate, and unrelated."
>
> — Mary Baratta-Lorton (cited on p.112 of *About Teaching Mathematics* by Marilyn Burns)

Children in the primary grades develop an awareness of patterns in their environment. Those who are successful in mathematics expand this awareness into understanding and apply it to learning about the number system. Children who do not look for patterns as a means of understanding and learning mathematics often find mathematics to be quite difficult. Thus, it is critical in the early grades to establish an early predisposition to looking for patterns, creating patterns, and extending patterns.

Children should **recognize, construct and extend patterns** with pattern blocks, cubes, toothpicks, beans, buttons and other concrete objects. Children in kindergarten can recognize patterns in motion, color, designs, sound, rhythm, music, position, sizes, and quantities. They are very aware of sound and rhythm, and can clap out patterns that repeat, such as clap-clap-clap-pause, clap-clap-clap-pause, etc. They can sit in a circle and wear colored hats which make a pattern, such as red-white-blue, red-white-blue. One child can walk around the circle and tap successive children in an arm-shoulder-head pattern. The teacher may ask the class who the next person to be tapped on the head would be if the pattern were to be continued. In addition to repeating patterns, students should have experiences with expanding patterns. They can indicate such a pattern by using motion: skip-jump-turn around, skip-jump-jump-turn around, skip-jump-jump-jump-turn around, and so on. Songs are excellent examples of repetition of melody or of words, as well as of rhythmic patterns. Children's literature abounds with stories which rely on rhythm, rhyming, repetition and sequencing. As students move on to first and second grade, they should start to create their own patterns and develop pictorial and symbolic representations of those patterns. The transition will be from working with patterns using physical objects to using pictures, letters, and geometric figures in two and three dimensions, and then to using symbols, such as words and numbers, to represent patterns.

Categorization and classification are also important skills for students in the primary grades. Kindergartners should have numerous opportunities to sort, classify, describe, and order collections of many different types of objects. For example, students might be asked to sort attribute shapes, buttons, or boxes into two groups and explain why they sorted them as they did. This area offers an excellent opportunity for students to verbalize their thought processes and to integrate learning in mathematics and science as they sort natural objects such as shells, rocks, or leaves.

Discovering a rule and **input-output** games are two other settings in which primary children can enhance

their work and their skills with patterns. The children might be asked to solve the *mystery of the crackers* as the teacher slowly and deliberately gives every boy two crackers and every girl four crackers one day during snack time. The inequity is addressed, of course, as soon as the children solve the mystery by discovering the rule that the teacher was using. On a different day, first graders can be told that they may request between 3 and 5 crackers for snack. But then each child is actually given two crackers less than his or her request. Again, as soon as the children verbalize the relationship between the request (input) and the portion allotted (output), they receive the missing crackers.

Establishing the habit of looking for patterns is exceedingly important in the primary grades. By studying patterns, young children develop necessary tools to become better learners of mathematics as well as better problem solvers. In addition, patterns help students to appreciate the beauty of mathematics and to make connections within mathematics and among mathematics and other subject areas.

Standard 11 — Patterns, Relationships, and Functions — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and in grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Reproduce, extend, create, and describe patterns and sequences using a variety of materials.**
    - Students make a collage with examples of patterns in nature.
    - Students create visual patterns with objects, colors, or shapes using materials such as buttons, macaroni, pattern blocks, links, cubes, attrilinks or attribute blocks, toothpicks, beans, or teddy bear counters. They challenge other students to describe or extend their patterns.
    - Students sort objects such as leaves, buttons, animal pictures, and blocks, using categories corresponding to characteristics like number of holes, number of sides, shapes, or thickness.
    - One child walks around the outside of a circle and taps successive children in a head-shoulder-shoulder-head pattern. The teacher asks who the next person to be tapped on the head would be if the pattern were to be continued. The children sing and act out the song, *Head, shoulders, knees and toes*.
    - Students describe patterns made from circles, triangles, and squares, and select the next shape in the pattern.
    - Students make patterns with letters and extend the sequence.
    - As an assessment task, students use letters to translate patterns they have created with objects — for example, RRBRRB for a Unifix pattern of red-red-blue-red-red-blue, or ABBCABBC for a shape pattern of square - circle - circle - triangle - square - circle - circle - triangle.
    - Students connect the dots to make a picture by following a number sequence, such as 2, 4, 6, 8, ...
    - Students create *one more* and *one less* patterns.
    - Students create patterns with the calculator. They enter any number such as 10, and then add 1 for 10+1= = = ... The calculator will automatically repeat the function and display 11, 12, 13, 14, etc. Some calculators may need to have the pattern entered twice: 10+1=+1= = = ... Other calculators will need 1++10= = = ... Students may repeatedly add or subtract any number.
    - Students name things that come in pairs (or 4s or 5s): eyes, ears, hands, arms, legs, mittens, shoes, bicycle wheels, etc. They work in pairs to find how many people there are if there are 20 eyes.
    - Students count by 2, 5, or 10 using counters or creating color patterns with Unifix or Linker

cubes; they repeat this using skip counting on a number line.

- Students use skip counting or calculators to find multiples of numbers and then color them on the hundreds chart. Linking cubes or Unifix cubes can be used to build towers or trains with every other cube or every third cube a certain color to illustrate, recognize, and practice skip counting patterns.
- Students write their first name repeatedly on a 10x10 grid, and then color the first letter of their name to create a pattern. They discuss the patterns formed.
- Students identify the same pattern in a variety of contexts. For example, black-white-black-white is like sit-stand-sit-stand and ABAB and up-down-up-down and straight-curve-straight-curve.
- Students identify patterns on a calendar using pictures or numerals. For example, in November, even dates might be marked with a snowflake, and odd dates with a picture of a turkey. Or, they might mark each date with the day of the week.
- Students create a pattern using various rubber stamp blocks or picture designs.
- Students use or create patterns with geometric figures (circles, triangles, squares, pentagons, hexagons, etc.) and record how many of each shape exist after each repeating cluster.
- Students create a mosaic design (tessellation) made of different shapes using objects such as pattern blocks. They color congruent shapes of a mosaic design with the same color.

2. Use tables, rules, variables, open sentences, and graphs to describe patterns and other relationships.

- Students complete a table given several starting numbers and a verbal rule.
- Kindergartners look at *Anno's Counting House* by Mitsumasa Anno to see if they can figure out the pattern that is used in moving from one set of pages to the next. The people in this book move, one by one, from one house to another.
- Students describe the pattern illustrated by the numbers in a table by using words (e.g., one more than), and then the teacher helps them to represent it with symbols in an open sentence (□ = ♡ + 1).
- Students use colored squares to make a graph showing the multiples of 3 and relate this to a table and an expression involving a variable, such as 3 x □.

3. Use concrete and pictorial models to explore the basic concept of a function.

- Students study the pictures in *Anno's Math Games II* by Mitsumasa Anno. As they do, they try to figure out what happens to the objects as the elves put them into the magic machine. Sometimes the number of objects doubles, sometimes the objects grow eyes, and sometimes the objects turn into circles.
- Students put numbers into *Max the Magic Math Machine* and read what comes out. (The teacher acts as Max.) Then they describe what Max is doing to each number. The teacher pays careful attention to the students' responses to assess their levels of understanding.
- Students investigate a hole-making machine that puts 4 holes into buttons. They make a table that shows the number of buttons put into the machine and the total number of holes

that must be made in them. Then they write a sentence that describes how the total number of holes changes as new buttons are added.

- Students play *Guess my Rule*. The teacher gives them a starting number and the result after using the rule. She continues giving examples until students discover the rule.
- Students count the number of pennies (or nickels) in 1 dime, 2 dimes, 3 dimes and record their results in chart form. They study the patterns and discuss the *rules* observed.
- Students consider the cost of two or three candies if one candy costs one dime. They make a chart using the information.
- Students count the number of lifesavers in an assorted pack. They make a table showing the number of each color and the total number in one pack. Then, assuming all of the packs are the same, they make a table showing the total number of each color for 2 packs, 3 packs, 4 packs, and so on. They check their results with packs of lifesavers, which in general, have the same number of each color.

4. Observe and explain how a change in one physical quantity can produce a corresponding change in another.

- Students discuss how ice changes to water as it warms. They talk about how it snows in January or February but rains in April or May.
- Students plant seeds and watch them grow. They write about what they see and measure the height of their plants as time passes. They discuss how changes in time bring about changes in the height of the plants. They also talk about how other factors might affect the plants, such as light and water.

5. Observe and recognize examples of patterns, relationships, and functions in other disciplines and contexts.

- Students go on a *pattern hunt* around the classroom and the school, discussing the patterns they find.
- Students sing and act out songs like "Rattlin' Bog" (Bird on the leaf, and the leaf on the tree, and the tree in the hole, and the hole in the ground, . . .) and "Old MacDonald Had a Farm."
- In reading, students recognize patterns in rhythm, in rhyming, in syllables and in sequencing. Stories such as *Ten Black Dots* by Donald Crews, *Five Little Monkeys Jumping on a Bed* by Eileen Christelow, *Jump, Frog, Jump* by Robert Kalan, *The Little Red Hen*, and Dr. Seuss books offer such opportunities. Visual patterns can be shown using picture representations for children's books such as *1 Hunter* by Pat Hutchins, *Rooster's Off to See the World* by Eric Carle, *The Patchwork Quilt* by Valerie Flournoy, and *The Keeping Quilt* by Patricia Polacco.
- Students identify every third letter of the alphabet; every fourth letter, etc. They use those sets of letters to see what words they can make.
- Students choose a day. Using a calendar, they identify the name of the next day, of the previous day, and also the name of the day two days (or more) before and after. They select a date, and give the date of the next day and of the previous day, the name of the month, of the next month, and of the previous month. They give the name of the date two days before

and after, and three days (or more) before and after.

- Students graph daily weather patterns, showing sunny, cloudy, rainy or snowy days. Then they discuss monthly or seasonal patterns.
- In social studies, students identify traffic patterns such as how many cars, trucks, or buses pass the front of the school during five minutes at different times of the day. They keep records for five days, organizing the information in chart form.
- In art, students observe patterns in pictures, mosaics, tessellations, and Escher-like drawings, as well as in wallpaper, fabric, and floor tile designs.

6. Form and verify generalizations based on observations of patterns and relationships.

- Students draw pictures of faces and make a table that shows the number of faces and the number of eyes. The teacher writes a sentence on the board that the class composes, describing the patterns that they find.
- Students observe that there are 12 eggs in a carton of eggs. These are called a dozen. They explain how to find the number of eggs in 2 cartons, 3 cartons, and so on.
- Students write a sentence or more telling about the patterns they have observed in a particular activity. They may use pictures to describe or generalize what they have observed. For example, after students have colored multiples of a certain number on the hundreds chart, they write about the geometric pattern they observe on the chart.

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Seuss, Dr. Most Dr. Seuss books exhibit appropriate patterns.

The Little Red Hen. Many versions are available.

On-Line Resources

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additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 12 — Probability and Statistics — Grades K-2

Overview

Students can develop a strong understanding of probability and statistics from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in probability for early elementary students, as identified in the K-12 Overview, are **probability terms**, **the concept of the probability of an event,** and **predicting and determining probabilities.** In statistics they key components for early elementary students are **data collection, organization, and representation.**

The understanding of probability and statistics begins with their introduction and use at the earliest levels of schooling. Children are natural investigators and explorers — curious about the world around them, as well as about the opinions and the habits of their classmates, teachers, neighbors and families. Thus, a fertile setting already exists in children for the development of statistics and probability skills and concepts. As with most of the curriculum at these grade levels, the dominant emphasis should be experiential with numerous opportunities to use the concepts in situations which are real to the students. Statistics and probability can and should provide rich experiences to develop other mathematical content and relate mathematics to other disciplines.

Kindergarten students can **gather data** and **make simple graphs** to organize their findings. These experiences should provide opportunities to look for patterns in the data, to answer questions related to the data, and to generate new questions to explore. By playing games or conducting experiments related to chance, children begin to develop an understanding of probability terms.

First- and second-grade children should continue to **collect and organize data**. These activities should provide opportunities for students to have some beginning discussions on **sampling**, and to **represent their data** in charts, tables, or graphs which help them **draw conclusions,** such as *most children like pizza* or *everyone in the class has between 0 and 4 sisters and brothers,* and raise new questions suggested by the data. As they move through this level, they should be encouraged to design data collection activities to answer new questions. They should be encouraged to see how frequently statistical claims appear in their life by collecting and discussing appropriate items from advertising, newspapers, and television reports.

Students in these grades should experience probability at a variety of levels. Numerous children's games are played with random chance devices such as spinners and dice. Students should have opportunities to play games using such devices. Games where students can make decisions based upon their understanding of probability help to raise their levels of consciousness about the significance of probability. Gathering data can lead to issues of probability as well. Students should experience **probability terms** such as *possibly*, *probably*, and *certainly* in a variety of contexts. Statements from newspapers, school bulletins, and their own experiences should highlight their relation to probability. In preparation for later work, students need to have experiences which involve systematic listing and counting of possibilities, such as all the possible outcomes when three coins are tossed (see Standard 14, Discrete Mathematics.)

Learning probability and statistics provides an excellent opportunity for connections with the rest of the mathematics standards as well as with other disciplines. Probability provides a rich opportunity for children to begin to gain a sense of fractions. Geometry is frequently involved through use of student-made spinners

of varying-sized regions and random number generating devices such as dice cubes or octahedral (eight-sided) shapes. The ability to explain the results of data collection and attempts at verbal generalizations are the foundations of algebra. Making predictions in both probability and statistics provides students opportunities to use estimation skills. Measurement using non-standard units occurs in the development of histograms using pictures or objects and in discussions of how the frequency of occurrence for the various options are related. Even the two areas of this standard are related through such things as the use of statistical experiments to determine estimates of the probabilities of events as a means for solving problems such as how many blue and red marbles are in a bag.

The topics that should comprise the probability and statistics focus of the kindergarten through second grade mathematics program are:

collecting data
organizing and representing data with tables, charts and graphs
beginning analysis of data using concepts such as range and “most”
drawing conclusions based on data
using probability terms correctly
predicting and determining probability of events

Standard 12 — Probability and Statistics — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Formulate and solve problems that involve collecting, organizing, and analyzing data.**
    - Students collect objects such as buttons, books, blocks, counters, etc. which can be sorted by color, shape, or size. They classify the objects and color one square of a bar graph for each item using different colors for each category. Then they compare the categories and discuss the relationships among them.
    - As an assessment following activities such as the one described above, young students are given a sheet of picture stickers and a blank sheet of paper. They sort the stickers according to some classification scheme and then stick them onto the paper to form a pictograph showing the number in each category.
    - At the front of the room is a magnetic board and, for every child in the class, a magnet with that child's picture. At the start of each day, the teacher has a different question on the board and the children place their magnet in the appropriate area. It might be a bar graph tally for whether they prefer vanilla, chocolate or strawberry ice cream or a Venn diagram where students place their magnet in the appropriate area based on whether they have at least one brother, at least one sister, at least one of both, or neither.
    - Students survey their classmates to determine preferences for things such as food, flavors of ice cream, shoes, clothing, or toys. They analyze the data collected to develop a cafeteria menu or to decide how to stock a store.
    - Second graders record and graph the times of sunrise and sunset one day a week over the entire year. They calculate the time from sunrise to sunset, make a graph of the amount of daylight, and interpret these weekly results over the year.
    - A second grader, upset because she had wanted to watch a TV show the night before but had to go to bed instead, asks the teacher if the class can do a survey to find out when most children her age go to bed.

2. **Generate and analyze data obtained using chance devices such as spinners and dice.**
    - Students roll a die, spin a spinner, or reach blindly into a container to select a colored marble, with replacement, a dozen times. They then color the appropriate square in a bar graph for each pick. *Did some results happen more often or less often than others? Do you think some results are more likely to happen than others?* They repeat the experiment, this time without replacement, and compare the results.
    - Students spill out the contents of cups containing five two-colored counters and record the number of red sides and the number of yellow sides. They perform the experiment twenty

times, examine their data, and then discuss questions such as *Does getting four red sides happen more often than two red sides?* They explain their reasoning.

- Each student has a 4-section spinner. Working in pairs, the students spin their spinners simultaneously and together they record whether they have a match. After doing this several times, they predict how many times they would have a match in 20 spins. Then they compare their prediction with what happens when they actually spin the spinners 20 times. They repeat the activity with a different number of equal sections marked on their spinners. Students in the second grade combine the results of all the students in the class, and compare their predictions with the class total.

3. Make inferences and formulate hypotheses based on data.

- Students roll a pair of dice 100 times and make a frequency bar graph of the sums. They compare their results with those of their classmates. *Do your graphs look essentially alike? Which sum or sums came up the most? Does everyone have a 'winning' sum? Is it the same for everyone? Why do some sums come up less than others?*
- Children are regularly asked to think about their data. *Is there a pattern in the dice throws, bean growth, weather, temperature, or other data? What causes the patterns? Are the patterns in their data the same as those of their classmates?*

4. Understand and informally use the concepts of range, mean, mode, and median.

- When performing experiments, children are regularly asked to find the largest and smallest outcomes (range) for numerical data and the outcome that appeared most often (mode). They are asked to compare the mode they obtained for an experiment with the modes found by their classmates.

5. Construct, read, and interpret displays of data such as pictographs, bar graphs, circle graphs, tables, and lists.

- After collecting and sorting objects, children develop a pictograph or histogram showing the number of objects in each category.
- Students design and make tallies and bar graphs to display data on information such as their birth months.
- Students list all possible outcomes of probability experiments, such as tossing a penny, nickel, and dime together.
- Working in cooperative groups, students are given six sheets of paper each containing an outline of a circle which has been divided into eight equal sectors. The students color each whole circle a different color and then cut their circles into individual sectors so each group has 8 sectors in each of 6 colors. Then they roll a die eight times keeping a tally of the results using orange for rolls of 1, blue for rolls of 2, and so on. They use these eight colored sectors to record their results in a circle graph, which they put aside. They repeat this twice and get two other circle graphs. Finally, as a whole class activity, they gather the circle graphs from all the groups, and rearrange the sectors to make as many solid color circles as they can. They discuss the results.
- Students regularly read and interpret displays of data; they also read information from their classmates' graphs and discuss the differences in their results.

6. Determine the probability of a simple event, assuming equally likely outcomes.

- Children roll a die ten times and record the number of times each number comes up. They combine their tallies and discuss the class results.
- Children predict how often heads and tales come up when a coin is tossed. They toss a coin ten times and tally the number of heads and tails. *Are there the same number of heads and tails?* They combine their tallies and compare their class results with their predictions. (See *Making Sense of Data*, in the *Addenda Series*, by Mary Lindquist.)

7. Make predictions that are based on intuitive, experimental, and theoretical probabilities.

- Second graders are presented with a bag in which they are told are marbles of two different colors, twice as many of one color as the other. They are asked to guess the probability for drawing each color if a single marble is drawn. *Is this the same as flipping a coin? Will one color be picked more often than the other?* The experiment is performed repeatedly and tallies are recorded. The chosen marble is returned to the bag each time before a new marble is drawn. The children discuss whether their estimates of the probabilities made sense in light of the outcome.
- Students are told that a can contains ten beads, some red ones, some yellow ones, and some blue ones. They are asked to predict how many beads of each color are in the can. The students attempt to determine the answer by doing a statistical experiment. One at a time, each child in the class draws a bead, records the color with a class tally, and replaces it. At various times in the process, the teacher asks the children to return to their prediction to determine if they want to modify it.
- As an informal assessment of the students' understanding of these concepts, they are presented with a bag in which they are told there are 10 yellow marbles and 2 blue ones. They are asked to predict what color marble they will pick out of the bag if they pick without looking, and about how many students in the class will pick a blue marble.

8. Use concepts of certainty, fairness, and chance to discuss the probability of actual events.

- Students work through the *Elevens Alive!* lesson that is described in the Introduction to this *Framework*. They make number sentences adding up to 11 by dropping 11 chips which are yellow on one side and red on the other, and writing 11= 4+7 when four chips land yellow-side-up and seven chips land red-side-up. They notice that they are writing some number sentences more frequently than others, and these observations lead into a discussion of probability.
- Each child plants five seeds of a fast growing plant. They count the number of seeds which sprout and discuss how many seeds might sprout if they had each planted ten, or twenty, or a hundred seeds. They explain their reasoning. (The numbers can be adjusted for different grade levels.)
- Students predict how many M&Ms of each color are in a large unopened mystery bag. To help make these predictions, cooperative groups are given a handful of M&Ms from the bag; they tally the count of the colors, report their results, and prepare graphs of their results. Students refine their predictions by looking at the class totals. The mystery bag is then opened and the colors counted. Students discuss how their prediction matches the actual

count and how the experiment helped them make their prediction.

- Students examine various types of raisin bran cereal. They experiment with scoops of cereal and determine the number of raisins that appear in each scoop. They make inferences about which brand might be the "raisiniest."

References

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Burton, G., et al. *Kindergarten Book. Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades K-6.* Reston, VA: National Council of Teachers of Mathematics, 1991.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

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Standard 13 — Algebra — Grades K-2

Overview

Students can develop a strong understanding of algebraic concepts and processes from consistent experiences in classroom activities where a variety of manipulatives and technology are used. The key components of this understanding in algebra, as identified in the K-12 Overview, are: **patterns, unknown quantities, properties, functions, modeling real-world situations, evaluating expressions** and **solving equations and inequalities.**

Students begin their study of algebra in grades K-2 by learning about the use of pictures and symbols to represent variables. They look at **patterns** and describe those patterns. They begin to look for **unknown numbers** in connection with addition and subtraction number sentences. They **model** the relationships found in real-world situations by writing number sentences that describe those situations. At these grade levels, the study of algebra is very much integrated with the other content standards. Children should be encouraged to play with concrete materials, describing the patterns they find in a variety of ways.

People tend to learn by identifying patterns and generalizing or extending them to some conclusion (which may or may not be true). A major emphasis in the mathematics curriculum in the early grades should be the opportunity to experience numerous **patterns**. The development of algebra as a language should build on these experiences. The ability to extend patterns falls under Standard 11 (Patterns and Functions), but having students communicate their reasoning is also an algebra expectation. Initially, ordinary language and concrete materials should be used for communication. As students grow older and patterns become more complex, students should develop the ability to use tables and pictures or symbols (such as triangles or squares) to represent numbers that may change or are **unknown** (variable quantities).

The primary grades provide an ideal opportunity to lay the foundation for the development of the ability to represent situations using **equations or inequalities** (open sentences) and solving them. Students can be asked to communicate or represent relationships involving concrete materials. For example, two students might count out eight chips and place them on a mat. One of the students then places a margarine tub over some of the counters and challenges the other student to figure out how many chips are hidden under the tub. A more complex situation might involve watching the teacher balance a box and two marbles with six marbles. The students draw a picture of the situation, and try to decide how many marbles would balance the box by physically removing two marbles from each side of the balance. In a problem involving an inequality, students might be asked to find out how many books Jose has if he has more than three books but fewer than ten. Situations from the classroom and the students' real experiences should provide ample opportunities to construct and solve such open sentences.

As operations are developed, students need to examine **properties** and make generalizations. For example, giving students a set of problems which follow the pattern *3 + 4, 4 + 3, 1 + 2, 2+1*, etc. should provide the opportunity to develop the concept that order does not affect the answer when adding (the commutative property). After students understand that these properties are not necessarily true for all operations (e.g., *5 – 2* is not equal to *2 – 5*), the teacher should mention that the properties are important enough to be given names. However, the focus of this work should be on using the properties of operations to make work easier rather than on memorizing the properties and their names.

Students in grades K-2 spend a great deal of time developing meaning for the arithmetic operations of addition, subtraction, multiplication, and division. As they work toward understanding these concepts, they focus on developing **mathematical models** for concrete problem situations. The number sentences that they write to describe these problem situations form a foundation for more sophisticated mathematical models.

Standard 13 — Algebra — Grades K-2

Indicators and Activities

The cumulative progress indicators for grades K-2 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and in grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Understand and represent numerical situations using variables, expressions, and number sentences.**

    - Students represent a problem situation with an open sentence. For example: *If there are 25 students in the class and Marie brought 26 cookies for snack, how many will be left over? (26 – 25 = ?)* Another example might be: *We have 10 cups left in the package and there are 25 children in the class, so how many more cups do we need to get? (10 + ? = 25)*
    - Students read *The Doorbell Rang* by Pat Hutchins. They act out the story and realize that many different combinations of students can share 12 cookies equally.
    - Students make a table relating the number of people and the number of eyes. They use a symbol such as a stick figure to represent the number of people and a cartoon drawing of an eye to represent the number of eyes and then express the relationship between them.

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2. **Represent situations and number patterns with concrete materials, tables, graphs, verbal rules, and number sentences, and translate from one to another.**

    - Students in groups are given a container to which they add water until its height is 5 centimeters, measured with Cuisennaire rods. They add marbles to the container until the height of the water is 6 centimeters. They continue adding marbles, recording each time the number of marbles it takes to raise the water level one centimeter. They describe the relationship between the number of marbles added and the height of the water.
    - In regular assessment activities, students look at a series of pictures which form a pattern. They draw the next shape, describe the pattern in words, and explain why they chose to draw that shape.
    - Using a calculator, students play *Guess My Rule*. The lead student enters an expression such as *5+4* and presses the = key; she shows only the answer to her partner. The second student tries to guess the rule by entering different numbers, one at a time, pressing the = key after each number. The calculator, after each = is pressed, should show the sum of the entered number and the second addend (in this case, 4). (Some calculators perform this function differently; see the user's manual for instructions.) When the second student thinks she knows the pattern (in this case, *adding 4*), she makes a guess. The pattern is written in words and then as a rule using a picture or symbol for the variable (the number which the second student enters).

- Placing four different-colored cubes in a can, students predict which color would be drawn out most often if each child draws one cube without looking. The teacher helps the students keep track of their results by making a chart with the colors on the horizontal axis and the number of times a color is drawn on the vertical axis. As students select cubes, an "x" is placed above the color drawn, forming a frequency diagram. After several turns, the students describe the patterns they see in the graph.
- Students read *Ten Apples Up on Top!* by Theo Le Sieg and discuss the mathematical comparisons and equations that appear in the story.

3. Understand and use properties of operations and numbers.

- Students are given five computational problems to solve. They are permitted to use the calculator on only two of them. Two of the problems are related to another two by operation properties (e.g., *3 + 2* and *4 + 6* are related to *2 + 3* and *6 + 4* by the commutative property) and the last involves a property of number such as adding *0*. Students share their thought processes in a follow up discussion.
- The second grade teacher has a box containing slips of paper with open sentences such as *25 – 8* = □ or 15 + □ = 23. Students draw out a slip and tell or write a story which would involve a situation modeled by the sentence.
- Students discover that, since the order of the numbers when adding them is not important, they can solve a problem like *3 + 8* by starting with 8 and counting up 3, as well as by starting with 3 and counting up 8.
- In their math journals, students write their reactions to the following situation:

  Sally just used her calculator to find out that 324 + 486 was equal to 810. In another problem, she must find the answer to 486 + 324. What should she do? Why?

4. Construct and solve open sentences (example: 3 + □ = 7) that describe real-life situations.

- Kindergarten students play the *hide the pennies* game. The first player places a number of pennies (say 7) on the table and lets the other player count them. The first player covers up a portion of the pennies, and the second player must determine how many are covered. They may represent the situation with markers or pictures to help them. Some second-grade students are ready to write a number sentence that describes the situation.
- Students are given a bag with Unifix cubes. They are told that the bag and 2 cubes balance 7 cubes. They use a balance scale to find how many cubes are in the bag.

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Le Sieg, Theo. *Ten Apples Up on Top!* New York, NY: Random House, 1961.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards.*

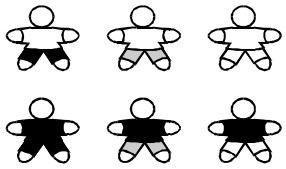
Standard 14 — Discrete Mathematics — Grades K-2

Overview

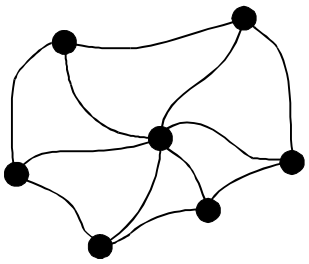
The five major themes of discrete mathematics, as discussed in the K-12 Overview, are **systematic listing, counting, and reasoning; discrete mathematical modeling using graphs (networks) and trees; iterative (that is, repetitive) patterns and processes; organizing and processing information;** and **following and devising lists of instructions, called "algorithms," and using them to find the best solution to real-world problems**.

Despite their formidable titles, these five themes can be addressed with activities at the K-2 grade level which involve purposeful play and simple analysis. Indeed, teachers will discover that many activities they already are using in their classrooms reflect these themes. These five themes are discussed in the paragraphs below.

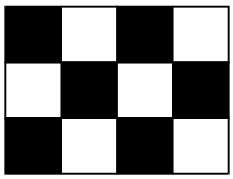
Activities involving **systematic listing, counting, and reasoning** can be done very concretely at the K-2 grade level. For example, dressing cardboard teddy bears with different outfits becomes a mathematical activity when the task is to make a list of all possible outfits and count them; pictured on the right are the six outfits that can be arranged using one of two types of shirts and one of three types of shorts. Similarly, playing any game involving choices becomes a mathematical activity when children reflect on the moves they make in the game.

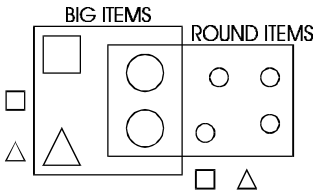


An important **discrete mathematical model** is that of a **network** or **graph**, which consists of dots and lines joining the dots; the dots are often called *vertices (vertex* is the singular) and the lines are often called *edges*. (This is different from other mathematical uses of the term "graph.") The two terms "network" and "graph" are used interchangeably for this concept. An example of a graph with seven vertices and twelve edges is given below. You can think of the vertices of this graph as islands in a river and the edges as bridges. You can also think of them as buildings and roads, or houses and telephone cables, or people and handshakes; wherever a collection of things are joined by connectors, the mathematical model used is that of a network or graph. At the K-2 level, children can recognize graphs and use life-size models of graphs in various ways. For example, a large version of this graph, or any other graph, can be "drawn" on the floor using paper plates as vertices and masking tape as edges. Children might select two "islands" and find a way to go from one island to the other island by crossing exactly four "bridges." (This can be done for any two islands in this graph, but not necessarily in another graph.)



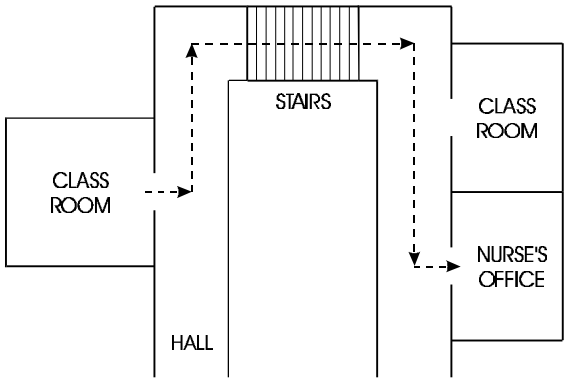
Children can recognize and work with **repetitive patterns and processes** involving numbers and shapes, using objects in the classroom and in the world around them. For example, children at the K-2 level can create (and decorate) a pattern of triangles or squares (as pictured here) that cover a section of the floor (this is called a "tessellation"), or start with a number and repeatedly add three, or use clapping and movement to simulate rhythmic patterns.





Children at the K-2 grade levels should investigate ways of **sorting items** according to attributes like color, shape, or size, and ways of **arranging data** into charts, tables, and family trees. For example, they can sort attribute blocks or stuffed animals by color or kind, as in the diagram, and can count the number of children who have birthdays in each month by organizing themselves into birthday-month groups.

Finally, at the K-2 grade levels, children should be able to **follow and describe simple procedures and** determine and discuss **what is the best solution** to a problem. For example, they should be able to follow a prescribed route from the classroom to another room in the school (as pictured below) and to compare various alternate routes, and in the second grade should determine the shortest path from one site to another on a map laid out on the classroom floor.



Two important resources on discrete mathematics for teachers at all levels are the 1991 NCTM Yearbook *Discrete Mathematics Across the Curriculum K-12* and the 1997 DIMACS Volume *Discrete Mathematics in the Schools: Making An Impact.* Another important resource for K-2 teachers is *This Is MEGA-Mathematics!*

Standard 14 — Discrete Mathematics — Grades K-2

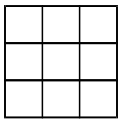
Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Explore a variety of puzzles, games, and counting problems.**

   - Students use teddy bear cut-outs with, for example, shirts of two colors and shorts of three colors, and decide how many different outfits can be made by making a list of all possibilities and arranging them systematically. (See illustration in K-2 Overview.)
   - Students use paper faces or Mr. Potato Head type models to create a "regular face" given a nose, mouth, and a pair of eyes. Then they use another pair of eyes, then another nose, and then another mouth (or other parts) and explore and record the number of faces that can be made after each additional part has been included.
   - Students read *A Three Hat Day* and then try to create as many different hats as possible with three hats, a feather, a flower, and a ribbon as decoration. Students count the different hats they've made and discuss their answers.
   - Students count the number of squares of each size (1 x 1, 2 x 2, 3 x 3) that they can find on the square grid below. They can be challenged to find the numbers of small squares of each size on a larger square or rectangular grid.



   - Students work in groups to figure out the rules of addition and placement that are used to pass from one row to the next in the diagram below, and use these rules to find the numbers in the next few rows.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

     In this diagram, called Pascal's triangle, each number is the sum of the two numbers that are above it, to its left and right; the numbers on the left and right edges are all 1.

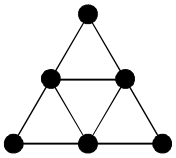
   - Students cut out five "coins" labeled 1¢, 2¢, 4¢, 8¢, and 16¢. For each number in the counting sequence 1, 2, 3, 4, 5, ... (as far as is appropriate for a particular group of students), students determine how to obtain that amount of money using a combination of different

coins.

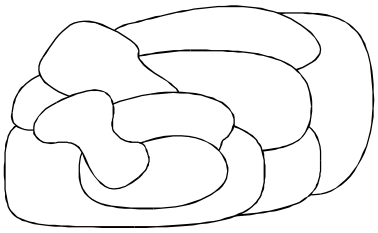
- Students play simple games and discuss why they make the moves they do. For example, two students divide a six-piece domino set (with 0-0, 0-1, 0-2, 1-1, 1-2, and 2-2) and take turns placing dominoes so that dominoes which touch have the same numbers and so that all six dominoes are used in the chain.

2. Use networks and tree diagrams to represent everyday situations.

- Students find a way of getting from one island to another, in the graph described in the K-2 Overview laid out on the classroom floor with masking tape, by crossing exactly four bridges. They make their own graphs, naming each of the islands, and make a "from-to" list of islands for which they have found a four-bridge-route. (Note: it may not always be possible to find four-bridge-routes.)
- Students count the number of edges at each vertex (called the **degree** of the vertex) of a network and construct graphs where all vertices have the same degree, or where all the vertices have one of two specified degrees.
- On a pattern of islands and bridges laid out on the floor, students try to find a way of visiting each island exactly once; they can leave colored markers to keep track of islands already visited. Note that for some patterns this may not be possible! Students can be challenged to find a way of visiting each island exactly once which returns them to their starting point. Similar activities can be found in *Inside, Outside, Loops, and Lines* by Herbert Kohl.



- Students create a map with make-believe countries (see example below), and color the maps so that countries which are next to each other have different colors. *How many colors were used? Could it be done with fewer colors? with four colors? with three colors? with two colors?* A number of interesting map coloring ideas can be found in *Inside, Outside, Loops and Lines* by Herbert Kohl.

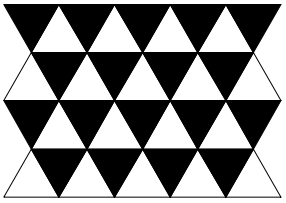


3. Identify and investigate sequences and patterns found in nature, art, and music.

- Students use a calculator to create a sequence of ten numbers starting with zero, each of which is three more than the previous one; on some calculators, this can be done by pressing 0 + 3 = = = . . . , where = is pressed ten times. As they proceed, they count one 3, two 3s,

three 3s, etc.

- Students "tessellate" the plane, by using groups of squares or triangles (for example, from sets of pattern blocks) to completely cover a sheet of paper without overlapping; they record their patterns by tracing around the blocks on a sheet of paper and coloring the shapes.

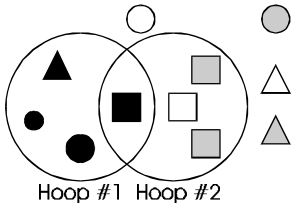


- Students listen to or read *Grandfather Tang's Story* by Ann Tompert and then use tangrams to make the shape-changing fox fairies as the story progresses. Students are then encouraged to do a retelling of the story with tangrams or to invent their own tangram characters and stories.
- Students read *The Cat in the Hat* or *Green Eggs and Ham* by Dr. Seuss and identify the pattern of events in the book. Students could create their own books with similar patterns.
- Students collect leaves and note the patterns of the veins. They look at how the veins branch off on each side of the center vein and observe that their branches are smaller copies of the original vein pattern. Students collect feathers, ferns, Queen Anne's lace, broccoli, or cauliflower and note in each case how the pattern of the original is repeated in miniature in each of its branches or clusters.
- Students listen for rhythmic patterns in musical selections and use clapping, instruments, and movement to simulate those patterns.
- Students take a "patterns walk" through the school, searching for patterns in the bricks, the play equipment, the shapes in the classrooms, the number sequences of classrooms, the floors and ceilings, etc.; the purpose of this activity is to create an awareness of all the patterns around them.

4. Investigate ways to represent and classify data according to attributes, such as shape or color, and relationships, and discuss the purpose and usefulness of such classification.

- Students sort themselves by month of birth, and then within each group by height or birth date. (Other sorting activities can be found in *Mathematics Their Way*, by Mary Baratta-Lorton.)
- Each student is given a card with a different number on it. Students line up in a row and put the numbers in numerical order by exchanging cards, one at a time, with adjacent children. (After practice, this can be accomplished without talking.)
- Students draw stick figures of members of their family and arrange them in order of size.
- Students sort stuffed animals in various ways and explain why they sorted them as they did. Students can use *Tabletop, Jr.* software to sort characters according to a variety of attributes.
- Using attribute blocks, buttons, or other objects with clearly distinguishable attributes such as color, size, and shape, students develop a sequence of objects where each differs from the previous one in only one attribute. *Tabletop, Jr.* software can also be used to create such sequences of objects.
- Students use two Hula Hoops (or large circles drawn on paper so that a part of their interiors

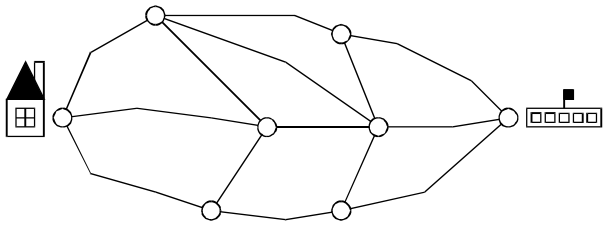
overlap) to assist in sorting attribute blocks or other objects according to two characteristics. For example, given a collection of objects of different colors and shapes, students are asked to place them so that all red items go inside hoop #1 and all others go on the outside, and so that all square items go inside hoop #2 and all others go on the outside. *What items should be placed in the overlap of the two hoops? What is inside only the first hoop? What is outside both hoops?*



This is an example of a Venn diagram. Students can also use Venn diagrams to organize the similarities and differences between the information in two stories by placing all features of the first story in hoop #1 and all features of the second story in hoop #2, with common features in the overlap of the two hoops. A similar activity can be found in the *Shapetown* lesson that is described in the First Four Standards of this *Framework*. *Tabletop, Jr.* software allows students to arrange and sort data, and to explore these concepts easily.

5. Follow, devise, and describe practical lists of instructions.

- Students follow directions for a trip within the classroom — for example, students are asked where they would end up if they started at a given spot facing in a certain direction, took three steps forward, turned left, took two steps forward, turned right, and moved forward three more steps.
- Students follow oral directions for going from the classroom to the lunchroom, and represent these directions with a diagram. (See K-2 Overview for a sample diagram.)
- Students agree on a procedure for filling a box with rectangular blocks. For example, a box with dimensions 4"x4"x5" can be filled with 10 blocks of dimensions 1"x2"x4". (Linking cubes can be used to create the rectangular blocks.)
- Students explore the question of finding the shortest route from school to home on a diagram like the one pictured below, laid out on the floor using masking tape, where students place a number of counters on each line segment to represent the length of that segment. (The shortest route will depend on the placement of the counters; what appears to be the most direct route may not be the shortest.)



- Students find a way through a simple maze. They discuss the different paths they took and

their reasons for doing so.

- Students use Logo software to give the turtle precise instructions for movement in specified directions.

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Software

Logo. Many versions of Logo are commercially available.

Tabletop, Jr. Broderbund Software. TERC.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

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Standard 15 — Conceptual Building Blocks of Calculus — Grades K-2

Overview

Students in the early primary grades bring to the classroom intuitive notions of the meaning of such terms as *biggest, largest, change*, and so forth. While they may not know the names of large numbers, they certainly have a sense of "largeness." The cumulative process indicators related to this standard for grades K-2 deal primarily with investigating patterns of growth and change over time.

Students in grades K-2 should investigate many different types of patterns. For some of these patterns, such as 2, 4, 6, 8, ... , the same number is added (or subtracted) to each number to get the next number in the sequence. When these patterns are represented with a bar graph, the tops of the bars can be connected by a straight line, so the pattern represents **linear growth.** Older students should also see patterns that grow more rapidly, such as 2, 4, 8, ... These growing patterns involve **exponential growth**; each number in the series is multiplied (or divided) by the same number to get the next one. In this situation, when the tops of the bars on a graph are connected, they do not form a straight line. These types of patterns can be investigated very easily by using calculators to do the computation; students enjoy making the numbers bigger and bigger by using a constant addend (e.g., *2* + *2* = = =) or a constant multiplier (e.g., *2* x *2* = = =). (Note that some calculators require different keystrokes to achieve this effect.) By relating these problems to concrete situations, such as the growth of a plant, students begin to develop a sense of **change over time**.

Students also begin to develop a sense of change with respect to **measurement**. Students begin to measure the length of objects by using informal units such as paperclips or Unifix cubes; they should note that it takes more small objects to measure a given length than large ones. By the end of second grade, they begin to describe the area of objects by counting the number of squares that cover a figure. Again, they should note that it takes more small squares to cover an object than it does large ones. They should also begin to investigate what happens to the area of a square when each side is doubled. Students also need to develop volume concepts by filling containers of different sizes. They might use two circular cans, one of which is twice as high and twice as wide as the other, to find that the large one holds eight times as much as the small one. Measurement may also lead to the beginnings of the idea of **a limiting value** for young children. For example, the size of a dinosaur footprint might be measured by covering it with base ten blocks. If only the 100 blocks are used, then one estimate of the size of the footprint is found; if unit blocks are used, a more precise estimate of the size of the footprint can be found.

Students in grades K-2 should also begin to look at concepts involving **infinity**. As they learn to count to higher numbers, they begin to understand that, no matter how high they count, there is always a bigger number. By using calculators, they can also begin to see that they can continue to add two to a number forever and the result will just keep getting bigger.

The conceptual underpinnings of calculus for students in grades K-2 are closely tied to their developing understanding of number sense, measurement, and pattern. Additional activities relating to this standard can be found in the chapters discussing these other standards.

Standard 15 — Conceptual Building Blocks of Calculus — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Investigate and describe patterns that continue indefinitely.**
    - Students model repeating patterns with counters or pennies. For example, they repeatedly add two pennies to their collection and describe the results.
    - Students create repeating patterns with the calculator. They enter any number such as 10, and then add *1* for *10 + 1 === . . .* . The calculator will automatically repeat the function and display 11, 12, 13, 14, etc. each time the = key is pressed. (Some calculators may need to have the pattern entered twice: *10 + 1 = + 1 === . . .* etc. Others may use a key sequence such as *1++10 ===. . .* .) Students may repeatedly add (or subtract) any number.
    - Second graders create a pattern with color tiles. They start with one square and then make a larger square that is two squares long on each side; they note that they need four tiles to do this. Then they make a square that is three squares long on each side; they need nine tiles to do this. They make a table of their results and describe the pattern they have found.
    - Students investigate a doubling (growing) pattern with Unifix cubes. They begin with one cube and then "win" another cube. Then they have two cubes and "win" two more. They continue this pattern, each time "winning" as many cubes as they already have. Repeating this process, they begin to see how quickly the number of cubes grows. They investigate this further using a calculator.
    - Students start with a rectangular sheet of paper that represents a cake. They simulate eating half of the cake by cutting the sheet in half and removing one of the halves. They eat half of what is left and continue this process. They describe the pattern, noting that after they repeat this about ten times, the cake is essentially gone.

2. **Investigate and describe how certain quantities change over time.**
    - Students keep a daily record of the temperature both inside and outside the classroom. They graph these temperatures and look at the patterns.
    - Students keep a monthly record of their height and record the data collected on a bulletin board. At the end of the school year, they describe what happened over time.
    - Students play catch with a ball in the school playground. One person counts out the number of times the ball is thrown, the other counts out the distance that it travels, a third person adds that distance to the total, and a fourth person records the totals. Afterwards they discuss how the total distance changes over time; they recognize that the same amount is added repeatedly.
    - Students study the changes in the direction and length of the shadow of a paper groundhog at

different times of the day. They relate these observations to the position of the sun (e.g., as the sun gets higher, the shadow gets shorter).

- Students discuss how ice changes to water as it gets hotter. They talk about how it snows in January or February but rains in April or May.
- Students plant seeds and watch them grow. They write about what they see and measure the height of their plants at regular time intervals. They discuss how changes in time result in changes in the height of the plant. They also talk about how other factors might affect the growth of the plant, such as light and water.

3. **Experiment with approximating length, area, and volume, using informal measurement instruments.**
    - Students measure the width of a bookcase using the 10-rods from a base ten blocks set. They record this length (perhaps as 6 rods or 60 units). Then they measure the bookcase using ones cubes; some of the students decide that it is easier just to add some ones cubes to the 10-rods that they have already used. They find that the bookcase is actually closer to 66 units long. They decide that they can get a better estimate of length when they use smaller units.
    - Students use pattern blocks to cover a picture of a turtle. They count how many of each type of block (green triangle, yellow hexagon, etc.) they used. They make a bar graph that shows how many blocks each student used. They discuss why some students used more blocks than others and what they could do to increase or decrease the number of blocks used.
    - Students play with containers of various sizes, transferring water from one container to another. They note that it takes two cups of water to fill a small milk carton. A pitcher holds three milk cartons of water, but four milk cartons overflow the pitcher. Then they find that it takes seven cups to fill the pitcher even though three milk cartons is only six cups. They decide that the smaller container gives a better idea of how much the pitcher will hold.
    - Students find the area of huge dinosaur footprints that they find taped to the classroom floor. They first try to fit as many green 4" tiles as possible into a footprint without any overlapping, and without any tiles sticking out of the footprint. Before removing the green tiles, they cover them with blue 2" tiles, and count the number of blue 2" tiles used. Then they remove the green tiles and try to fit more blue 2" tiles into the footprint without overlapping; they discover that they can fit more and discuss why that is the case. They repeat this, using red 1" tiles. They notice that with smaller tiles, less of the footprint is uncovered, so that the smaller tiles provide a better estimate of the footprint's size.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 15 — Conceptual Building Blocks of Calculus — Grades 3-4

Overview

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THIS DOCUMENT IS A DISTILLATION OF NEW JERSEY'S CURRICULUM FRAMEWORKS, WHICH ARE MUCH LONGER DOCUMENTS. WE HAVE INCLUDED ONLY THE ACTIVITIES AND PORTIONS AIMED TOWARDS KINDERGARTEN STUDENTS. THE COMPLETE FRAMEWORKS CAN BE FOUND ON THE NJDOE'S WEBSITE:

HTTP://WWW.NJ.GOV/NJDED/FRAMEWORKS

A GUIDE ON HOW TO UTILIZE NJDOE'S MATH FRAMEWORKS TO IMPROVE TEACHING AND LEARNING

FOR A COPY OF THE CCCS, CPI, FRAMEWORK ACTIVITIES, SAMPLE TESTS, & SAMPLE QUESTIONS THAT CORRESPOND TO NEW JERSEY'S STANDARDIZED TESTS - VISIT THE NTU & NPS WEB PAGES *(http://www.ntuaft.com) & (http://www.nps.k12.nj.us)* &/OR NTU'S EDUCATION RESOURCE CENTER

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