Special Review Assessment (SRA)
High School Proficiency Assessment (HSPA)

Mathematics:
A Rubric Scoring Handbook

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MATHEMATICS:
A RUBRIC SCORING HANDBOOK

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INTRODUCTION

We want ALL students to achieve the standards.

The vision of the mathematics standards is focused on achieving one crucial goal:

To enable ALL of New Jersey’s children to acquire the mathematical skills, understandings, and attitudes that they will need to be successful in their careers and daily lives.

The vision of success for all students in mathematics depends on:
1. Establishing learning environments that facilitate student learning of mathematics.
2. A commitment to equity and to excellence.
3. Defining the critical goals of mathematics education today – what students should know and be able to do (i.e., content and processes).

New Jersey’s mathematics standards rest on the notion that an appropriate mathematics curriculum results from a series of critical decisions about three inseparably linked components: content, instruction, and assessment.

The Special Review Assessment (SRA)

New Jersey Statute (N.J.S.A. 18A:7c-3) and Administrative Code (N.J.A.C. 6A:8-4.1) provide for an alternate assessment of students who have met all graduation requirements except for demonstrating proficiency in all content areas of the High School Proficiency Assessment (HSPA). The Special Review Assessment (SRA) process is the designated alternate assessment for the HSPA. The SRA provides students with the opportunity to show their proficiency of the HSPA knowledge and skills in a familiar setting.

The SRA is aligned to the HSPA test specifications to ensure that students who demonstrate proficiency through the SRA have demonstrated the same knowledge, skills, and performance levels as students who are proficient on the HSPA itself. The process used for the development of the SRA is designed to ensure that the SRA is fair, reliable, and comparable for all eligible SRA candidates, including students from special populations.

The Special Review Assessment consists of mathematics and language arts literacy Performance Assessment Tasks (PATs) that are administered to individual students.

High school students have 3 chances to demonstrate proficiency on the HSPA – the spring of their junior year, the fall of their senior year, and the spring of their senior year. For every administration of the HSPA, each school district receives Individual Student Reports (ISRs) for their students. These reports give specific information about proficiency results for each student who took the HSPA.

A student whose scaled score is below 200 (partially proficient) in one or more HSPA content areas and is expected to complete all state and local graduation requirements in the twelfth grade is eligible for the SRA process.
Each SRA question, known as a Performance Assessment Task (PAT), consists of one to five open-ended parts derived from a common stem. Each PAT is aligned to the HSPA content specifications by clusters. The clusters for mathematics with associated macros are:

I. Number Sense, Concepts, and Applications
   A. Understand types of numbers, our numeration system, and the ways they are used and applied in real-world situations.
   B. Apply ratios, proportions, and percents to a variety of situations.

II. Spatial Sense and Geometry
   A. Recognize, visualize, analyze, and apply geometric properties, relationships, and patterns in real-world and/or problem-solving contexts using models, manipulatives, and technology.
   B. Use coordinate geometry in problem-solving situations and apply the principles of congruence, similarity, and transformations.
   C. Apply the principles of measurement and geometry to solve problems involving direct and indirect measurement.

III. Data analysis, Probability, Statistics, and Discrete Mathematics
   A. Determine, interpret, and use probabilities of simple and compound events.
   B. Understand and interpret statistical distributions and apply to real-world situations.
   C. Collect, organize, represent, analyze, and interpret data.
   D. Apply the concepts and methods of discrete mathematics to model and explore a variety of practical situations.
   E. Use iterative and recursive patterns and processes to model a variety of practical situations and solve problems.

IV. Patterns, Functions, and Algebra
   A. Recognize, create, and extend a variety of patterns and use inductive reasoning to understand and represent mathematical and other real-world phenomena.
   B. Use various types of functions to represent mathematical or real-world situations.
   C. Use algebraic concepts and processes to concisely express, analyze, and model real-world situations.
SCORING STUDENT RESPONSES

Scoring rubrics provide the criteria for evaluating and scoring student performance. Item-specific rubrics, which are individual scoring guides, are supplied for each SRA PAT for mathematics. These item-specific rubrics are based upon the generic mathematics rubric and are developed by a committee of mathematicians and teachers. Rubrics ensure that there is consistency, fairness, and accuracy in scoring open-ended questions.

GENERIC MATHEMATICS SCORING RUBRIC

3-Point Response
The response shows complete understanding of the problem’s essential mathematical concepts. The student executes procedures completely and gives relevant responses to all parts of the task. The response contains few minor errors, if any. The response contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made.

2-Point Response
The response shows nearly complete understanding of the problem’s essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response may have minor errors. The explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences.

1-Point Response
The response shows limited understanding of the problem’s essential mathematical concepts. The response and procedures may be incomplete and/or may contain major errors. An incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made.

0-Point Response
The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand why and how decisions were made.

Two content-certified mathematics panel members must independently score each SRA PAT. The two independent ratings must be equal (agreeing) or within one point of each other (contiguous/adjacent). The final score for the mathematics PAT is the mean score of the two independent scores. Mathematics proficient score is a mean score of 2 on each PAT which means that the two SRA panel members each scored the student’s response a 2 or better.

However, if the two SRA panel members’ scores disagree by more than one point, a third content-certified panel member must score the response. The new PAT score is derived by taking the mean of the two highest contiguous scores. If no two of the three scores are in agreement, the student must complete another PAT.
Scoring Student Responses in a Language Other than English

For SRA mathematics PATs, students’ responses in their native language can only be scored by two independent certificated secondary mathematics teachers. If the certificated secondary mathematics teachers are not fluent in the native language, certificated world language teachers, certificated ESL teachers or certificated bilingual teachers may orally translate the student responses for the SRA panel members. The teacher involved in the oral translation will be considered a non-scoring member of the SRA Mathematics Panel and should be indicated as such on the HSPA SRA Student Profile Form.

Recommendations for Scoring Student Responses

In order for all students to receive fair, accurate, and consistent scores, there are some recommended procedures for the SRA Mathematics Scorers to consider.

1) Take time to answer the PAT during/after the administration of the PAT to the student. This will allow the scorer to obtain insight as to which questions are more challenging to the student or where a student may misinterpret one of the questions.

2) Compare your answer to that given as a sample response. Note that the sample response is just one way of answering the PAT. There may be many acceptable responses.

3) Become familiar with the scoring rubric. The item specific rubric cannot identify all the parameters for every student response. It is important for the scorer to also keep the generic rubric in mind when scoring the student’s PAT. Does the student response demonstrate insufficient understanding (0 points), a limited understanding (1 point), nearly complete understanding (2 points), or complete understanding (3 points) of the PAT’s essential mathematical concepts?

4) Before scoring a student’s PAT, it is important that both scorers fully understand the rubric. When one scorer gives the student a score of 1, and the other scorer gives the student a score of 2, a misinterpretation of either the student response or the scoring rubric has arisen.

5) A student’s explanation (justification) for a question may be in the form of an algebraic expression, a table, diagrams, charts, sentences/paragraphs, mathematical equations, etc.

6) A student may take 2 PATs from the same macro if he/she has exhausted all of the PATs from the other Macros within that Cluster. For example, if a student is taking 2 PATs from Cluster 1, that student should successfully pass a PAT from Macro A and Macro B. However, if a student passed a PAT from Macro A and could not pass any of the PATs from Macro B, that student can take another PAT from Macro A to successfully pass that Cluster.

7) Please familiarize yourself with the SRA Administration Manual.

8) If you have a question about a PAT or have found an error, please get in touch with Timothy Giordano, Mathematics Coordinator at the NJ Department of Education (609/633-8015 or timothy.giordano@doe.state.nj.us).
HSPA/SRA MATHEMATICS

SAMPLE PATs and STUDENT RESPONSES

The mathematic PATs and responses selected to appear in this handbook were completed by twelfth-grade students who participated in the October 2002 HSPA/SRA Pilot Test. The responses appear as the students wrote them; no corrections have been made other than the deletion of specific names of teachers, administrators, students, schools, and districts.

The score point for each student response is noted. Please feel free to use these materials as practice materials for students as well as practice materials for scorers.
“Dorothy is running for president.”
Dorothy is running for president of the student body and wants to create campaign posters to hang throughout the school. She has determined that there are four main hallways that need six posters each. A single poster takes one person 30 minutes to create and costs a total of $1.50.

A) What would be the total cost for Dorothy to create all the needed posters? Show your work.

B) If two people working together can create a poster in 20 minutes, how much total time would Dorothy save by getting a friend to help her? Show your work.

C) If Dorothy works alone for 3 hours, and is then joined by her friend, calculate exactly how much total time it will take to create all the necessary posters. Show your work.

D) Omar, Dorothy’s opponent, decided to create his posters on a Saturday and get his friends Janice and Beth to help. He knows that he can create 24 posters in 12 hours if he works alone. He also knows that Janice can create 24 posters in 10 hours and Beth can create 24 posters in 9 hours. How long will it take them, if all three of them work together to create the 24 posters? Round all decimals to the nearest hundredths. Show your work.

E) When Omar went to purchase his posters, he discovered that the cost of creating a poster had increased by 20%. How many posters will he be able to create if he wants to spend the same amount of money on his posters as Dorothy? Justify your answer.

Materials/Resources:
Calculator
HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric B0705-001R for this PAT.
Sample Response:

A) \((1.50) \times (4 \times 6) = 36\)

B) \((30 \times 24) - (20 \times 24) = 240\) minutes \(= 4\) hours
   or \(24 \times 10\) (time saved per poster) \(= 240\) minutes
   \((3)(4) - 2(4) = 12 - 8 = 4\) hours

C) One person can make 6 posters in 3 hours. The remaining 18 posters take 2 people 360 minutes or 6 hours. 3 hours + 6 hours = 9 hours total.

D) \(\frac{h}{12} + \frac{h}{10} + \frac{h}{9} = 1\)
   \(.29h = 1\)
   \(h = 3.45\) hours (note: used rounding process twice)
   \(h = 3.40\) hours (note: used rounding process only once)
   OR
   \(180\left(\frac{h}{12} + \frac{h}{10} + \frac{h}{9}\right) = (180)\)
   \(15h + 18h + 20h = 180\)
   \(53h = 180\)
   \(h = \frac{180}{53}\) hours = \(3 \frac{21}{53}\) hours
   or 3.40 hours

E) \((1.20)(1.50) = 1.80\) per poster
   \(\$36 \div 1.80 = 20\) posters
   Omar can create 20 posters.
3 Points  The student found the correct total cost for creating the posters, the amount of time saved if a friend helps, and the correct total time needed to create all posters in part C. The total time for Omar to create 24 posters is correctly calculated in part D, and “20” is the correct answer for part E. Work was shown for all parts.

2 Points  The student found three or four correct answers out of five, and supported those answers with appropriate work. The other answers were either omitted, incorrect due to a process error, or not supported by work.

1 Point  The student found one or two correct answers out of five, supported with appropriate work. OR
The student found at least three correct answers, but no work was shown.

0 Points  The response shows an insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.
1.  $6 \times 4 = 24$
   $24 \times \$4 = \$96$

2. 2 people 1 person in 20 minutes
   24 480 = 8 hours 12-

   4 person 1 person 30 minutes 8
   24 720 = 12 hours 4 hours

3. 1 person 2 persons 180 minutes 180 : 30 = 6

   720 min. 12 hours
   180 min. 3 hours
   540 = 9 hours

   Jenice 24 persons in 10 hours 2.4
   Perk 24 persons in 9 hours 2.4
   Omar 24 persons in 12 hours 2

Score Point 2
The student found three correct answers, and supported those answers with appropriate work.
Score Point 2
The student found four correct answers, and supported those answers with appropriate work.
4 \times 6 = 24

$36.00

720

\frac{720}{800} = 0.9

\text{She would need 300 min with a friend working at 30 min each.}

\text{If she for 3 hours alone and does 6 posters, } 6 \times 3 = 18

\text{that leaves 18 posters left, the boyfriend } \frac{18}{2} = 9

\text{comes to help and they then complete 9 more every 60 min, then they complete 9}

\text{posts each in -180 min, changing partly working a total of 6 hours all together}

Omar: 3.5

Janice: 3.4

Beth: 3.6

\text{If Omar only did 7 since he works the slowest and Beth picked up}

\text{his other poster giving her 9 posters and}

\text{Janice did 8 they would all be done in 3 and 3/4 hours.}
Score Point 1
The student found one correct answer supported with appropriate work. The student has a limited understanding of the problem’s essential mathematical concepts.
Score Point 0
The response shows an insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
4 hallways - 6 posters in each hallway

1 poster = 30 min to create cost (1) $1.50

A) There are 4 hallways that need 6 posters each.

So you would multiply 6 x 4 which would equal 24 posters.

Now you would again multiply this time 24 which is the number of posters by the amount of each which is $1.50 your answer

$36 would be the total for Dorothy to create all the needed posters.

6 x 4 = 24

24 x $1.50 = $36

B) 1. 30 - (10 x 30) 20
2. 30 - (10 x 20) 20
3. 30 - (10 x 20) 20
4. 30 - (10 x 20) 20
5. 30 - (10 x 20) 20
6. 30 - (10 x 20) 20

if Dorothy did not have a second friend it would have taken her 3 hours to complete 6 posters. But now that she has a second friend it will reduce the 3 hrs into 2 hrs because it takes 30 minutes per each poster.

30 minutes per each poster

20 - 6 = 130

60 / 130 = 2 hrs.

C) Dorothy will be done in 3 hrs.

poster (1) 30 + (2) 30 = 60 1 hr. Dorothy would have 0 (zero) poster
poster (3) 30 + (4) 30 = 60 1 hr. to finish because she would already
poster (5) 30 + (6) 30 = 60 1 hr. be done.

Total 3 hrs. She would not need anymore time to

0.00 because there won't be anymore poster
Score Point 1
The student found two correct answers supported with appropriate work.
cluster 1/Macro B
PAT - 80705 COIS
10 No. 29

A) Hallways  posters  cost  poster

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>6</th>
<th>1.50</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

X = 24 posters  \( \times 36 \) = 36 dollars

*The total cost for Dorothy to create all the needed posters is 36 dollars

B) w/2 people  time/min.  w/1 person  time/min.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>20</th>
<th></th>
<th>1</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td></td>
<td>X</td>
<td></td>
<td>29</td>
<td>X</td>
</tr>
</tbody>
</table>

\( X = 180 \text{ minutes} \)  \( \times 720 \text{ minutes} \)

980/60 = 8 hrs  \( \div \) 720/60 = 12 hrs

*Dorothy would save 4 hrs by getting a friend to help her

C) Poster  \( \times / \) \( \times \) 1 2 3 4 5 6

|    | 30m, 1hr, 1.50 hrs, 12 hrs \( 2 \times 3 \) | 18 | X |

\( X = 360 \text{ min} \)

360/60 = 6 hrs
c) They will take to do all the necessary posters 6 hrs working both together.

\[
\begin{array}{ccc}
\text{posters} & \text{hrs} \\
D) & 24 & 12 & Omar & 2 \\
& 24 & 10 & Janice & 2.9 \\
& 24 & 9 & Beth & 2.7 \\
\hline & & & & 7.1
\end{array}
\]

If they worked together they will finish in 7 hrs. to create the 24 posters.

\[
\begin{align*}
1.50 \times 20 &= 30 \\
0.3 &+ 0.3 \\
1.80 \times 20 &= 36 \text{ dollars.}
\end{align*}
\]

Omar will be able to buy just 20 posters this is the only way he could spend the same amount as Dorothy.

---

**Score Point 2**
The student found three correct answers, and supported those answers with appropriate work.
A. 4 hallways \times 6 \text{ posters} \times \frac{24}{24} = 36 \text{ posters}

There are 4 hallways that need 6 posters each so I multiplied 6 times 4 to get 24. Then it costs $1.50 to make a poster, so I multiplied $1.50 times 24 to get $36.00.

B. She saves 4 hours because for every 2 she did on her own she did 3 with her friend.

C. It will take 9 hours because Dorothy could do a poster in 1/2 an hour and in three hours of making posters she could finish it. Then the remaining six hours they got 18 posters done & each one took 20 minutes to make.
D) By the information given Omar can make 2 in 1 hour, Janice could do 3 in 1 hour, and Beth can make 3.33 in 1 hour. All together in 1 hour they could make 8.33 posters in 1 hour.

\[
\begin{align*}
8.33 \\
+8.33 \\
\hline
16.66 \\
+8.33 \\
\hline
24.99
\end{align*}
\]

*Conclusion - it would take 3 hours to finish.*

E) If Dorothy spent $1.50 a poster and had 24 posters it would cost her $36.00 to pay for them all. Omar’s price has increased by 20%. That makes each poster .30¢ more than they originally were so Omar could get 20 posters for $36.00.

Score Point 2
The student found four correct answers, and supported those answers with appropriate work.
Score Point 2
The student found four correct answers, and supported those answers with appropriate work.
Interest rate
A bank offers an interest rate of \( r \) compounded \( n \) times per year. The formula for the amount of money, \( A \), in an account at the end of \( t \) years, is:

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

where \( P \) is the amount of money in the account at the beginning of the year (assuming no deposits or withdrawals).

A) If at the beginning of the year Joe had $1,000 in an account with 2% interest compounded semiannually, how much money would he have in the account at the end of the year? Show your work or provide an explanation for your answer.

B) The effective interest rate, \( R \), is the percent increase in the account over one year. What is the effective interest rate for Joe’s account? (Do not round your answer.) Show your work or provide an explanation for your answer.

C) Joe had \( x \) dollars in his account at the beginning of the year. Describe how to determine the amount of money Joe would have in his account after 1 year using the effective rate you found above.

Materials/Resources:
Calculator
HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric B0706-002R for this PAT.
Sample Response

A) **$1,020.10**

\[ 1,000 \left(1 + \frac{0.02}{2}\right)^2 = 1,020.10 \]

B) 0.0201 OR 2.01%  

\[ \frac{20.10}{1,000} = 0.0201 \]

C) \( x \times 1.0201 \)

OR

\( x + x \times 0.0201 \)

OR

Multiply the amount of money he has at the beginning of the year by 1.0201.
3 Points  The response contains correct values for parts A and B (showing work) and a clear description, using either words or an expression, of the method that would yield a correct value for part C. Since the numbers used are simple to work with, there should be no computation errors.

2 Points  The response contains correct answers for two of the three parts of the problem, including work or explanations as called for in the problem.
  OR
  There is a computation or copying error, not a process error, that leads to logical, but incorrect responses in other parts of the problem.

1 Point  One part of the problem is correctly done with work shown or explanations as required.
  OR
  The response contains two correct answers without work or explanation.
  OR
  The response contains correct answers for all three parts, but shows no work or explanations for parts A and B.

0 Points  The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.
A) $A = P \left(1 + \frac{r}{1}\right)^t = 1000 \left(1 + \frac{0.02}{1}\right)^1 = 1000 \left(1.02\right)^1 = 1000 \times 1.02 = 1020$

B) $2\% \times 2 = 0.04$

C) Joe had X dollars, so the problem would read like this: 
$A = X \left(1 + \frac{0.02}{1}\right)^1$. So divide r and 1, then add 1. Now you square it by 1. After all that you then times it by $X$, and that's how you get A, the answer.

Score Point 0
The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
$1,000 \times \frac{2\%}{20} = \frac{1,000 \times 0.02}{20} = \frac{20}{1,020} = \text{Amount of money}

$1,020 \times \frac{2\%}{20.40} = \frac{1,020 \times 0.02}{20.40} = \frac{20.40}{1,040.40}

$1,040.40 \times \frac{2\%}{20.81} = \frac{1,040.40 \times 0.02}{20.81} = \frac{20.81}{1,061.21}

$1,061.21 \times \frac{2\%}{21.22} = \frac{1,061.21 \times 0.02}{21.22} = \frac{21.22}{1,082.43} = \text{money}

B) The effective interest rate would be 8% per year:

$1,082.43 \times \frac{8\%}{80.59} = \frac{1,082.43 \times 0.08}{80.59} = \frac{80.59}{1,169.02}

\[ \begin{align*}
\text{(1)} & \quad \frac{1,000 \times 2\%}{20} = \frac{1,000 \times 0.02}{20} = \frac{20}{980} = \text{money} \\
\text{(2)} & \quad \frac{980 \times 2\%}{19.60} = \frac{980 \times 0.02}{19.60} = \frac{19.60}{960.40} = \text{money}
\end{align*} \]
Score Point 0
The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
Score Point 2
The response contains correct answers for two of the three parts of the problem with work provided. Part C) contains a flaw and therefore does not warrant a score point of 3.
\[ A = P \left( 1 + \frac{r}{n} \right)^{nt} \]

\[ A = 1000 \left( 1 + \frac{.02}{2} \right)^{2(1)} \]

\[ A = 1000 \left( 1 + .01 \right)^2 \]

\[ A = 1000 \left( 1.01 \right)^2 \]

\[ = 1000 \left( 1.0201 \right) \]

\[ = 1020.1 \]

B) 31.9, I put 1020.1 then percanted it.

C)
Quadrilaterals and slopes
Quadrilateral ABCD is graphed below with A(–3, 2), B(0, –2), C(8, 4), and D(5, 8).

A) Calculate the slopes of each side of ABCD and of the two diagonals. Show your work and label your responses.

B) Explain mathematically how you know that the quadrilateral is or is not each of the following types of quadrilaterals:
   1. parallelogram
   2. rectangle
   3. rhombus
   4. square

Materials/Resources:
Calculator
HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric A1103-004R for this PAT.
Sample Response:

A) \[ \text{slope of AB} = \frac{2 - (-2)}{-3 - 0} = \frac{4}{-3} = -\frac{4}{3} \]
\[ \text{slope of DC} = \frac{4 - 8}{8 - 5} = \frac{-4}{3} = -\frac{4}{3} \]
\[ \text{slope of BC} = \frac{-2 - 4}{0 - 8} = \frac{-6}{-8} = \frac{3}{4} \]
\[ \text{slope of AD} = \frac{2 - 8}{-3 - 5} = \frac{-6}{-8} = \frac{3}{4} \]
\[ \text{slope of AC} = \frac{2 - 4}{-3 - 8} = \frac{-2}{-11} = \frac{2}{11} \]
\[ \text{slope of BD} = \frac{-2 - 8}{0 - 5} = \frac{-10}{-5} = 2 \]

B) 1) **ABCD is a parallelogram.**
AB and DC have equal slopes so AB is parallel to DC.
BC and AD have equal slopes so BC is parallel to AD.
ABCD is a parallelogram because if a quadrilateral has both pairs of opposite sides parallel, then it is a parallelogram.

2) **ABCD is a rectangle.**
The slopes of AB and BC are opposite reciprocals, so AB is perpendicular to BC.
(note: any pair of consecutive sides can be used here.)
ABCD is a rectangle because if a parallelogram has one right angle, then it is a rectangle.

3) **ABCD is NOT a rhombus.**
The slopes of AC and BD are not opposite reciprocals so AC is not perpendicular to BD. Since the diagonals of the parallelogram are not perpendicular, it is NOT a rhombus.

4) **ABCD is NOT a square.**
Since ABCD is not a rhombus it cannot be a square because to be a square a figure must be both a rectangle and a rhombus.
3 Points  The response contains correct answers to both parts with explanation provided for part B. In part A, the response determines that the slopes of AB and DC are both $\frac{4}{-3}$, the slopes of BC and AD are both $\frac{3}{4}$, the slope of AC is $\frac{2}{11}$, and the slope of BD is 2. In part B, the response determines that ABCD is a rectangle and a parallelogram, but is not a rhombus or a square, and includes acceptable explanations for each case.

Note: Among others, the response might show mathematical justifications using the following theorems:

- If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.
- If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- If each diagonal of a quadrilateral bisects opposite angles of a quadrilateral, then the quadrilateral is a rhombus.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a rhombus.
- If the diagonals of a quadrilateral are congruent, then the quadrilateral is a rectangle or a square.

2 Points  There is evidence of work shown for both parts. For part A, at least four slopes are correctly calculated, and for part B, at least two sections are correctly answered.

OR

Part A has six correct answers calculated and the student correctly explains mathematically the classifications of at least two of the quadrilaterals.

OR

Part B has four correctly answered sections and the student made an attempt to answer part A with some valid work shown.

1 Point  At least one correct slope is provided or one correct answer for part B is identified.

0 Points  The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.
B) The quadrilateral is a parallelogram because it has a base and the height forms a 90° angle. It's a rectangle because each corner of it forms 90° angles. It's not a rhombus. And it's not a square because square has all equal on all four sides and this quadrilateral is
A. \( A(-3,2), B(0, -2), C(8, 4), \) and \( D(5, 8) \)

\[ AB \quad B(-3,2), B(0, -2) \]

\[ \frac{y_1 - y_2}{x_1 - x_2} \quad = \quad \frac{2 - (-2)}{-3 - 0} \quad = \quad \frac{4}{-3} \quad = \quad \frac{-4}{3} \]

\[ BC \quad B(0, -2), C(8, 4) \]

\[ \frac{y_1 - y_2}{x_1 - x_2} \quad = \quad \frac{-2 - 4}{0 - 8} \quad = \quad \frac{-6}{-8} \quad = \quad \frac{3}{4} \]

\[ CD \quad C(8, 4), D(5, 8) \]

\[ \frac{y_1 - y_2}{x_1 - x_2} \quad = \quad \frac{4 - 8}{8 - 5} \quad = \quad \frac{-4}{3} \]

\[ DA \quad D(5, 8), A(-3, 2) \]

\[ \frac{y_1 - y_2}{x_1 - x_2} \quad = \quad \frac{8 - 2}{5 - (-3)} \quad = \quad \frac{6}{8} \quad = \quad \frac{3}{4} \]

Diagonal \( AC \quad C(-3,2), C(8,4) \)

\[ \frac{y_1 - y_2}{x_1 - x_2} \quad = \quad \frac{2 - 4}{-3 - 8} \quad = \quad \frac{-2}{-11} \quad = \quad \frac{2}{11} \]

Diagonal \( BD \quad B(0, -2), D(5, 8) \)

\[ \frac{y_1 - y_2}{x_1 - x_2} \quad = \quad \frac{-2 - 8}{0 - 5} \quad = \quad \frac{-10}{-5} \quad = \quad \frac{2}{1} \]

B. It is a parallelogram because the slopes of the lines \( AB \) and \( CD \) are the same and the lines \( BC \) and \( DA \) have the same slope and \( 2 \) lines that have the same
Score Point 2
Part A has six correct answers calculated and the student correctly explains mathematically the classifications of at least two of the quadrilaterals.
Quadrilateral $ABCD$ is graphed below with $A(-3,3)$ $B(0,-2)$ $(8,4)$ $D(5,8)$

1. Calculate the slopes of each side of $ABCD$ and of the two diagonals. Show your work and label your responses.
   
   $\begin{align*}
   &\text{Slope of } AD \parallel 12 \\
   &\text{Slope of } BC = 10 \\
   &x + y = 10
   \end{align*}$

2. Explain mathematically how you know that the quadrilateral is or is not each of the following types of quadrilaterals:
   
   1) Parallelogram.
   2) Rectangle.
   3) Rhombus.
   4) Square.

   It is a rectangle because have a different, equal sides.

**Score Point 0**

The student rewrote the problem as his/her response. The response shows insufficient understanding of the problem’s essential mathematical concepts.
“A rectangular board and its shadow.”
A rectangular board that measures 2 m wide and 10 m long is leaning against a wall, as shown below. The sun is shining directly above the board.
A) Sketch the shape of the shadow that the board makes on the ground.

B) If the sun’s rays are vertical, describe how moving the bottom edge of the board closer to the wall would affect the width and the length of the shadow on the ground.

C) How close to the wall should the bottom edge of the board be positioned so that the shadow of the board forms a square? Explain.

D) Find the height at which the top of the board touches the wall when the shadow on the ground is a square. Explain how you found your answer.

Materials/Resources:
Calculator
HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric A1106-003R for this PAT.
Sample Response:

A) Shadow will be a rectangle.

B) This would shorten the shadow’s length, but this would not affect the shadow’s width at all.

C) The shadow of the board will be in the shape of a square when the distance between the ramp’s base and the wall is the same as the ramp’s width of 2 meters.

D) 9.79 or 9.8 meters

\[10^2 - 2^2 = x^2\]
\[100 - 4 = 96\]
\[\sqrt{96} = x\]
\[x = 9.79 \text{ or } 9.8\]
3 Points  The response contains four correct responses with appropriate sketches, descriptions, and explanations as required.

2 Points  The response addresses all four parts of the questions correctly, but for one or two cases the explanations are incomplete or missing.
OR
Three parts of the questions are answered correctly with explanations as required. The fourth part may be missing or incorrect.
OR
Part D and one other part are answered completely and correctly.

1 Point  Either part A, B, or C of the question is addressed and answered completely and correctly.
OR
Two parts of the question are addressed with correct or partially correct explanations.

0 Points  The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.
31

A) The length would increase and the width would decrease. As the board moves up, the shadow will get skinner. If the board continues, there would be no shadow at all and the would be flat against the wall.

B) Since the width of the board is 2 m, all sides of a square are equal, the board should be about 2 m away from the wall. Which would the length 2 m & width 2 m for the shadow. The top of the wall would be at 8 m high. Since wall is 10 m and the length of the board is 10 m so subtract 10 m from 10 m, which leaves you with 2 m. Every time the board moves down a meter, you add one back to the wall until you get the bottom of the board to be 8 meters. Then you the 2 meters.
moved away the wall and add it the sum of the 12-10. Then you should get 4 so subtract that from the length of the wall 12m which leaves the height of board \[
\frac{-4m}{8m}\] at 8 meters high.

**Score Point 1**
Part C of the question is addressed and answered completely and correctly.
If you move the bottom board closer to the wall, the Sun would only shine on half of it because the board is blocking the Sun, so you would only have half a shadow.

You should stand it straight up. So that the shadow would be a complete square. Because if you take the board down, the wall might fall. So you could just stand it up against the wall and it would be OK.

First, you would have to find the length of everything.

wall = 48
wood = 24
shadow = 18

The wood to the top of
the wall is 8.

Score Point 0
The response shows insufficient understanding of the problem's essential mathematical concepts. The procedures, if any, contain major errors.
b) As the board moves closer to the wall, 
the shadow would most likely shrink 
because the area where the shadow 
would decrease. The width wouldn't be affected.

According to the diagram, the 
board is 10m long, and to my 
conclusion the board is split 
by the line making it 5m 
on one side and 5m meters on the other. 
In order to make a square, all sides 
must be equal. Since the width is 
2m, that would mean the bottom edge 
would have to be 2m away from...
Score Point 1
The student demonstrated a limited understanding of the problem’s essential mathematical concepts. The student’s response to part C is correct (the bottom edge would have to be 2m away from the wall since the width is 2m). Part B is somewhat correct, “the shadow would most likely shrink”; however, the student then states that the “width wouldn’t be affected as much.”
Score Point 2
The response shows nearly complete understanding of the problem’s essential mathematical concepts. The student executes nearly all procedures and gives relevant responses to most parts of the task. The response has minor errors (the width is not mentioned in part B) and the explanations detailing how the problem was solved are unclear (part D).
“Mr. Johnson’s science classroom’s fish tanks.”
Mr. Johnson’s science classroom has 4 fish tanks filled with the same type of fish. Each tank contains 4 fish that are the same age. One tank contains one-year-old fish, another tank contains two-year-old fish, another tank contains three-year-old fish, and the last tank contains four-year-old fish. The students measured the length of each fish and made the following chart comparing the age of each fish to the length of the fish in millimeters.

<table>
<thead>
<tr>
<th>AGE (in years)</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>4</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>LENGTH (in millimeters)</td>
<td>25</td>
<td>30</td>
<td>27</td>
<td>29</td>
<td>42</td>
<td>36</td>
<td>39</td>
<td>54</td>
<td>51</td>
<td>46</td>
<td>55</td>
<td>50</td>
<td>64</td>
<td>63</td>
<td>58</td>
<td>57</td>
</tr>
</tbody>
</table>

A) Make a scatter plot of the data from the table.

B) Sketch a line of best fit for the data.

C) Find the slope of the line of best fit for the data.

D) What meaning does the slope of the line of best fit have for this particular application?

E) Mr. Johnson’s students believe that one of the fish is in the wrong tank and therefore has the wrong age recorded. Circle the point on your scatter plot that represents this fish.

F) What would most likely be the correct age for this fish? Explain how you found your answer.

Materials/Resources:
- Calculator
- HSPA Mathematics Reference Sheet
- Graph Paper

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric B1312-005R for this PAT.
Sample Response:

A), B)

\[
\text{C) } 10, \text{ I had the points (1, 27) and (2, 37) on my line of best fit. The slope is } \frac{37 - 27}{2 - 1} = 10
\]

D) the number of millimeters the fish grow each year

OR

the yearly average growth rate of the fish

OR

the amount of growth per year
E) See graph above.

F) 3 years old. I sketched a line of best fit and the $x$-value for the $y$-value of 54 is 3.

OR

I noticed that the 3-year-old fish measured in the 50s.
3 Points The student correctly answers all of the question’s six parts, though there may be one or more minor errors. For part A, the scatter plot includes all 16 points (complete graphical precision is not necessary) and supports a positive relationship between age and length of the fish. In part B, the line is positively sloped and looks like it reasonably approximates the best fit line. In part C, the correct slope is found for the student’s line of best fit. In part D, the response states that the slope shows the average annual growth of fish, in millimeters, even though it may not include the word “average.” In part E, the circled point corresponds to the outlier, (2, 54). In part F, age 3 is correctly identified and an adequate explanation is given.

2 Points Either an attempt is made to answer all six parts of the problem, or correct solutions to four parts are presented. The scatter plot may be sloppily drawn, the slope is incorrectly found despite an attempt to do so, or there may be other minor errors in the response.

1 Point An attempt is made to answer at least four parts of the problem, and a correct solution to two parts is presented. There are major errors in the response, such as failing to correctly identify the relationship between length and age in part D, and/or drawing a best fit line that is flat or has a negative slope.

0 Points The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain several serious errors. The student has difficulty understanding the directions, there may be no explanation of the solution, or the explanation may not be understandable.
A) I did make the scatter plot of the data from the table.
B) I did the line sketch of best fit for the data.
C)
D) It is intersecting slope of the line.
E) The fish which is in the upper tank that Mr. Johnson's students believe is in Tank 4 is 54 mm long in 54.
F) I think that the most likely be the correct age for containing Tank 4 fish but would be 60. Because all number are mix in the length and in the millimeter, so that's why I think next number supposed to be 60.
Score Point 0
The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.

Data:

<table>
<thead>
<tr>
<th>Age in Year</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>4</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in millimeters)</td>
<td>65</td>
<td>30</td>
<td>22</td>
<td>49</td>
<td>39</td>
<td>54</td>
<td>37</td>
<td>45</td>
<td>53</td>
<td>40</td>
<td>64</td>
<td>69</td>
<td>84</td>
<td></td>
</tr>
</tbody>
</table>

For each:

When I subtract like 25 - 30 = 5 and so on like 30 - 27 = 3.

Then I multiply by the smaller of 55 and another like 1 x 25 = 25 but 3 x 45 = 75 and so on.
Score Point 1
The student has a limited understanding of the problem’s essential mathematical concepts. The student correctly drew the scatter plot, could not draw a line of best fit, did not attempt to find the slope of the line of best fit, could not identify the meaning of the slope of the line of best fit, correctly circled the point that represented the fish in the wrong tank, and correctly identified the correct age of that fish.
Score Point 2
Correct solutions to four parts are presented (A, B, E, and F). The slope was incorrectly found but there was an attempt to do so.
Score Point 2
An attempt is made to answer all six parts of the problem. The student demonstrated a nearly complete understanding of the essential mathematical concepts presented. There were minor errors in the response.
“Classroom networks”
The technology teacher would like to network 3 different classrooms (A, B, and C) and the media center, so that each classroom is connected to the media center and to each other. One connection runs both ways between classrooms, so once classroom A is connected to classroom B, B is also connected to A.

A) Altogether, how many connections will need to be made for the computer network to be completed between the 3 classrooms and the media center? List all the possible connections and draw a diagram showing all the connections.

B) After starting with 3 classrooms, more connections are added and by the end of the month there are 15 connections in all. All connections are made for each classroom before adding another. What is the total number of classrooms that are connected to each other and to the media center? List all the possible connections, or draw a diagram showing all the connections.
C) Study the table below and complete the duplicate table provided for your use on the following page. Write a formula or rule, in terms of \( n \), that represents the pattern used to determine the number of connections needed for \( n \) classrooms to be connected to the media center and each other. **Be sure to write your answers on the duplicate chart provided.**

<table>
<thead>
<tr>
<th># of classrooms</th>
<th># of connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
</tr>
<tr>
<td>6</td>
<td>?</td>
</tr>
<tr>
<td>7</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( n )</td>
<td>?</td>
</tr>
</tbody>
</table>

D) Use your formula from Part C to show the number of connections if 10 classrooms are to be networked.

**Materials/Resources:**
Calculator
HSPA Mathematics Reference Sheet

**Techniques for PAT Scoring:**
See Mathematics Item-Specific Rubric D1606-006R for this PAT.
USE THIS TABLE FOR PART C.

<table>
<thead>
<tr>
<th># of classrooms</th>
<th># of connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td></td>
</tr>
</tbody>
</table>
Sample Response:

A) 6 connections

AM, BM, CM, AB, AC, BC

OR

OR

OR
B)  5 classrooms


OR

```
    A
   / \  \
  B   C
   \  / \
    D--E
```

OR

```
    A
   / \  \
  B   C
   \  / \
    D--E
```

```
CLUSTER 3/MACRO D

C)

<table>
<thead>
<tr>
<th># of classrooms</th>
<th># of connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>•</td>
<td></td>
</tr>
<tr>
<td>•</td>
<td></td>
</tr>
<tr>
<td>•</td>
<td></td>
</tr>
<tr>
<td>•</td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>( ? )</td>
</tr>
</tbody>
</table>

If \( n \) is the number of classrooms, then the number of connections = \( n \) + the number of connections of \( n – 1 \) classrooms.

C) The number of connections in stage 10 = the number of connections in stage 9 + 10
\[
= 45 + 10 \\
= 55
\]
3 Points  The response contains correct answers to all four parts of the question. In parts A and B, answers of “6 connections, 5 classrooms” are supported by lists and diagrams. Note: The response may fail to identify “6 connections, 5 classrooms,” but if the lists and diagrams support these values, they will receive full credit. A variety of diagrams are acceptable, provided all connections are shown. In part C, the table is completed correctly, and a formula or rule for stage \( n \) is also correct. For part D, the answer (55) is derived.

2 Points  The response contains many elements of a “3,” but some answers may be incomplete or incorrect. For example, only three parts of the question are answered correctly and completely.

OR

Parts A and B are correct, but part C has errors in the table which affect the formula and value of the \( n \) stage (part D).

OR

All values are correct, but support is incomplete or missing. For example, part A, B, C, and D values are correct but no formula or rule is written in part C.

1 Point  The student may earn 1 point for any of the following:

- two correct answers without support
- one correct answer with support
- one correct diagram or list
- a correct table

0 Points  The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.
The technology teacher would like to network 3 different classrooms (A, B, and C) and the media center, so that each classroom is connected to the media center and to each other. One connection runs both ways between classrooms, so one classroom A is connected to classroom B, B is also connected to A.

Classroom A
Classroom B
Classroom C
Media Center

A) The number of connections that will need to be made for the computer network to be completed between the 3 classrooms and the media center are

<table>
<thead>
<tr>
<th>Classrooms</th>
<th>Media center ↔ Classrooms</th>
</tr>
</thead>
</table>

A→^b B→^c C→^a Media C→^b

B) After starting with 3 classrooms, more connections are added and by the end of the month there are 15 connections in all. All connections are made for each classroom before adding another. The total number of classroom that are connected to each other and the media center are

Classrooms
A→^b C→^a Media C→^b
B→^c D→^e

4 + 3 + 2 + 1 + 5 = 15
Total number of Classroom are 5
c) I was asked to write a formula or rule that represents the pattern used to determine the number of connections needed for \( n \) classrooms to be connected to the media center and each other.

<table>
<thead>
<tr>
<th># of classrooms</th>
<th># of connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2 + 4</td>
</tr>
<tr>
<td>3</td>
<td>6 + 6</td>
</tr>
<tr>
<td>4</td>
<td>12 + 8</td>
</tr>
<tr>
<td>5</td>
<td>20 + 10</td>
</tr>
<tr>
<td>6</td>
<td>30 + 12</td>
</tr>
<tr>
<td>7</td>
<td>42 + 14</td>
</tr>
<tr>
<td>8</td>
<td>56 + 16</td>
</tr>
<tr>
<td>9</td>
<td>72 + 18</td>
</tr>
<tr>
<td>10</td>
<td>90 + 20</td>
</tr>
<tr>
<td>11</td>
<td>110 + 23</td>
</tr>
<tr>
<td>12</td>
<td>132</td>
</tr>
</tbody>
</table>

c) The rule was to subtract the 2 classroom connection, and whatever the difference comes out to be, you add 2 to it, and just keep adding 2 and the number of connection of the previous classroom. The number of connection needed for the \( n \) classroom are 132.

---

D) I made a rule to find the number of connection if 10 classrooms are to be networked. The rule was subtract the number of connection from the 1st 2 classroom, like, \( 6 - 2 = 4 \), the difference was 4, and then I subtract the next 2 classrooms, like, \( 12 - 6 = 6 \), and I found out that it keep adding 2 too the difference it gets when subtracting the two classrooms, so I kept on adding 2 to the number of connection I get for the previous classroom, and I kept on adding two, and it came out to be that 10 classrooms will have 132 connection.

---

**Score Point 2**

Parts A and B are correct, but part C has errors in the table which affect the formula and value of the \( n \) stage (part D).
A) 6 connections: 1 connection from A to B, 1 connection from B to C, 1 connection A to C, 1 from media to A, 1 from media to B, 1 from media to C.

B) 5 total number of Classrooms

C) n + 3 = ?

Score Point 2
Parts A and B are correct.
Score Point 1
One correct diagram (A).
Score Point 0
The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
HSPA/SRA MATHEMATICS
Sample PAT #7
Cluster 4/Macro A
PAT-A1108-007S

Patterns, sequences & limits
Assume that the following table continues forever, to the $n^{th}$ row. (Do not write in the chart below. A duplicate chart has been provided for your use).

<table>
<thead>
<tr>
<th>Row 1</th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>Row 2</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{5}{2}$</td>
<td>$-2\frac{1}{2}$</td>
</tr>
<tr>
<td>Row 3</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{5}{3}$</td>
<td>$-2\frac{2}{3}$</td>
</tr>
<tr>
<td>Row 4</td>
<td>?</td>
<td>$\frac{5}{4}$</td>
<td>$-2\frac{3}{4}$</td>
</tr>
<tr>
<td>Row 5</td>
<td>$\frac{1}{5}$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Row 100</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Row $n$</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

A) Complete rows 4 and 5 in the duplicate chart provided.

B) What numbers belong in row 100? Write your answers in the chart provided.

C) Write expressions to determine the values of the $n^{th}$ row, for columns A, B, and C.

D) What number do the values of column A approach? What number do the values of column C approach? Explain your answers or show what the values approach on a number line to support your answer.

Materials/Resources:
Calculator
HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric A1108-007R for this PAT.
USE THIS CHART FOR PARTS A AND B.

<table>
<thead>
<tr>
<th>Row</th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{5\frac{1}{2}}{2}$</td>
<td>$-2\frac{1}{2}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{5\frac{1}{3}}{3}$</td>
<td>$-2\frac{2}{3}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{5\frac{1}{4}}{4}$</td>
<td>$-2\frac{3}{4}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{1}{5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Sample Response:

A) and B)

<table>
<thead>
<tr>
<th>Row</th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>\frac{1}{2}</td>
<td>\frac{5\frac{1}{2}}{2}</td>
<td>-2\frac{1}{2}</td>
</tr>
<tr>
<td>3</td>
<td>\frac{1}{3}</td>
<td>\frac{5\frac{1}{3}}{3}</td>
<td>-2\frac{2}{3}</td>
</tr>
<tr>
<td>4</td>
<td>\frac{1}{4}</td>
<td>\frac{5\frac{1}{4}}{4}</td>
<td>-2\frac{3}{4}</td>
</tr>
<tr>
<td>5</td>
<td>\frac{1}{5}</td>
<td>\frac{5\frac{1}{5}}{5}</td>
<td>-2\frac{4}{5}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>100</td>
<td>\frac{1}{100}</td>
<td>\frac{5\frac{1}{100}}{100}</td>
<td>-2\frac{99}{100}</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

C) Column A = \frac{1}{n}

Column B = \frac{1}{n} + 5

Column C = \frac{1}{n} - 3

OR
\[-2 + \frac{n - 1}{n}\]

OR

\[-2 + \frac{n - 1}{n} \text{ or } \frac{-2n - n + 1}{n}\]

OR

\[-\frac{3n + 1}{n}\]

OR

\[-3 + \frac{1}{n}\]

**Note:** These expressions may be put into the chart in part A.

D) As \(n\) gets larger, \(\frac{1}{n}\) approaches zero. Therefore, the values in column A approach zero. The values in column C approach –3.
3 Points The response addresses all four parts of the questions, arriving at correct values or expressions shown in the sample response. In part D, the values 0 and –3 are supported by an explanation.

2 Points The 2-point response contains many elements of the 3-point response, but something is incomplete, incorrect, or missing. Any one of the following would lower the score to a “2”:
- Two incorrect table entries in rows 4, 5, or 100 (parts A + B)
- An incorrect expression for row \( n \) (part C)
- An incorrect value in part D with an incomplete or missing explanation
- Responding to only 3 parts of the question, i.e., omitting one part completely

1 Point Only one part of question (A, B, C, or D) is completely correct, OR
There is some correct work in several parts, but not enough to qualify for a “2.”

0 Points The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the required solutions or the explanation may not be understandable. How decisions were made may not be readily understandable.
USE THIS CHART FOR PART A.

<table>
<thead>
<tr>
<th>Row</th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>1</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>Row 2</td>
<td>(\frac{1}{2})</td>
<td>(5\frac{1}{2})</td>
<td>-2(\frac{1}{2})</td>
</tr>
<tr>
<td>Row 3</td>
<td>(\frac{1}{3})</td>
<td>(5\frac{1}{3})</td>
<td>-2(\frac{2}{3})</td>
</tr>
<tr>
<td>Row 4</td>
<td>(\frac{3}{4})</td>
<td>(5\frac{1}{4})</td>
<td>-2(\frac{3}{4})</td>
</tr>
<tr>
<td>Row 5</td>
<td>(\frac{1}{5})</td>
<td>(5\frac{2}{5})</td>
<td>-2(\frac{4}{5})</td>
</tr>
<tr>
<td>Row 6</td>
<td>(\frac{1}{6})</td>
<td>(5\frac{3}{6})</td>
<td>-2(\frac{5}{6})</td>
</tr>
<tr>
<td>Row 100</td>
<td>(\frac{1}{100})</td>
<td>(5\frac{9}{100})</td>
<td>-2(\frac{99}{100})</td>
</tr>
<tr>
<td>Row n</td>
<td>(\frac{1}{n})</td>
<td>(5\frac{r}{n})</td>
<td>-2(\frac{r}{n})</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
0 &< \frac{3}{2} < 1 \\
-3 &< \frac{2}{3} < 0
\end{align*}
\]

Because column A approaches up to \(\frac{0}{100}\) and column C approaches up to -2 \(\frac{100}{101}\)

Score Point 2
Parts A and B are correct. Error in part C – row \(n\), column C. One incorrect value in part D.
Score Point 1
Part A has an error in row 5, column C. Part B has an error in row 100, column C. Part C has an error in row \( n \), column C. Part D was not attempted.
<table>
<thead>
<tr>
<th>Row 1</th>
<th>1</th>
<th>6</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 2</td>
<td>$\frac{1}{2}$</td>
<td>$5\frac{1}{2}$</td>
<td>$-2\frac{1}{2}$</td>
</tr>
<tr>
<td>Row 3</td>
<td>$\frac{1}{3}$</td>
<td>$5\frac{1}{3}$</td>
<td>$-2\frac{2}{3}$</td>
</tr>
<tr>
<td>Row 4</td>
<td>$\frac{1}{4}$</td>
<td>$5\frac{1}{4}$</td>
<td>$-2\frac{3}{4}$</td>
</tr>
<tr>
<td>Row 5</td>
<td>$\frac{1}{5}$</td>
<td>$5\frac{1}{5}$</td>
<td>$-2\frac{4}{5}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Row 100</td>
<td>$\frac{1}{100}$</td>
<td>$5\frac{1}{100}$</td>
<td>$-2\frac{99}{100}$</td>
</tr>
<tr>
<td>Row(n)</td>
<td>$\frac{1}{n}$</td>
<td>$5\frac{1}{n}$</td>
<td>$-2\frac{n-1}{n}$</td>
</tr>
</tbody>
</table>

For column A, I will be divided by the number of rows and for column C, -2 and $\frac{n-1}{n}$ will be the answers. A can approach and column C can approach any number as long as it has a limit at being...
less than -3. Both A and C are decreasing

Score Point 2
Parts A, B, and C are correct. Part D has an incorrect value.
<table>
<thead>
<tr>
<th>Row</th>
<th>Col A</th>
<th>Col B</th>
<th>Col C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>1</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>Row 2</td>
<td>1/2</td>
<td>5 1/2</td>
<td>-2 1/2</td>
</tr>
<tr>
<td>Row 3</td>
<td>1/3</td>
<td>5 1/3</td>
<td>-2 2/3</td>
</tr>
<tr>
<td>Row 4</td>
<td>1/4</td>
<td>5 1/4</td>
<td>-2 3/4</td>
</tr>
<tr>
<td>Row 5</td>
<td>1/5</td>
<td>5 1/5</td>
<td>-2 3/5</td>
</tr>
<tr>
<td>Row 100</td>
<td>99 1/2</td>
<td>99</td>
<td>98 1/2</td>
</tr>
<tr>
<td>Row n</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\text{A.} & \quad \frac{A}{99 1/2} \quad \frac{B}{99} \quad \frac{C}{98 1/2} \\
\text{B.} & \quad \frac{1}{(6) + 16D} = \frac{1}{166} \\
& \quad 2\left(\frac{1}{16}\right) + 16D = \frac{99}{2} + 1/3 \\
& \quad 3\left(-\frac{1}{16}\right) + 16D = \frac{99}{2} \\
\text{C.} & \quad n + 3 = \text{R} \\
& \quad n + 2 = \text{R} \\
& \quad n + 1 = \text{R}
\end{align*} \]
Score Point 0
The response shows insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
USE THIS CHART FOR PARTS A AND B.

<table>
<thead>
<tr>
<th></th>
<th>Column A</th>
<th>Column B</th>
<th>Column C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 1</td>
<td>1</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>Row 2</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{5}{2}$</td>
<td>$-2 \frac{1}{2}$</td>
</tr>
<tr>
<td>Row 3</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{5}{3}$</td>
<td>$-2 \frac{2}{3}$</td>
</tr>
<tr>
<td>Row 4</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{5}{4}$</td>
<td>$-2 \frac{3}{4}$</td>
</tr>
<tr>
<td>Row 5</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{5}{5}$</td>
<td>$-2 \frac{4}{5}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Row 100</td>
<td>$\frac{1}{100}$</td>
<td>$\frac{5}{100}$</td>
<td>$-2 \frac{49}{100}$</td>
</tr>
<tr>
<td>Row $n$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{1}{N} + S - 7 = -2 S_n
\]

\[
\frac{1-N+S-7}{N} = \frac{1}{N} + S - 7 - 2 = \frac{1}{N}
\]

\[
\left( \frac{1}{N} + S - 7 \right) = \sum_{n=0}^{\infty}
\]

\[
\frac{-2 \frac{1}{N} + 1}{100 - 1} = \frac{100 - 1}{G^2}
\]
Score Point 2
Parts A, B, and C are correct. Part D has an incorrect value.
“Sheila decides to wallpaper her basement.”
The length of the rectangular playroom in Sheila’s basement is 1 ½ times its width. The room has no windows and has a height of 8 feet. Sheila has decided to wallpaper the entire room and put a border over the wallpaper at the top around the room. To determine how much wallpaper is needed, she has to add together the area of each wall to get the total surface area of the room. The door to this room measures 2.5 feet wide by 6.5 feet high and will not need wallpaper.
A) Let $x$ be the width of the room. Write and simplify an equation in terms of $x$ to determine the total surface area ($S$) of the playroom’s walls.

B) The playroom is 12 feet wide. Each roll of wallpaper will cover approximately 56 square feet. The wallpaper has no pattern; therefore, it is not necessary to allow for matching patterns when calculating the amount of wallpaper to purchase. What is the least number of rolls Sheila will need to paper her room? Show your work.

C) Sheila plans to spend no more than $350.00 (ignore tax) on the wallpaper and border. The border costs $19.97 per 10-yard roll. The border will be pasted onto the wallpaper at the top of each wall with no overlap. Explain how to determine the most expensive wallpaper Sheila can buy while staying within her budget.

Materials/Resources:
- Calculator
- HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric C1006-008R for this PAT.
Sample Response:

A) \[ S = 8x + 8(1.5x) + 8x + 8(1.5x) - (2.5)(6.5) \]
\[ S = 8x + 12x + 8x + 12x - 16.25 \]
\[ S = 40x - 16.25 \]

B) \[ S = 40(12) - (2.5)(6.5) \]
\[ S = 480 - (2.5)(6.5) = 463.75 \text{ sq. ft.} \]
\[ 463.75 \div 56 \approx 8.28 \approx 9 \text{ rolls} \]
9 rolls Answer

C) First, Sheila has to determine how much border she needs. The perimeter of the room is \(12 + 18 + 12 + 18 = 60 \text{ feet, which is 20 yards.} \) She would need 2 rolls of border which costs $39.94. Subtract $39.94 from $350.00 and that leaves $310.06 to spend on wallpaper. If she needs 9 rolls, then \[ \frac{310.06}{9} \approx 34.45. \] Sheila can spend no more than $34.45 per roll on wallpaper and still be within her budget.
3 Points  The student correctly answers all three parts of the question. In part A, the student writes and simplifies an equation, leading to \( S = 40x - 16.25 \) as the answer.

In part B, the student uses the answer to part A, substituting the given value (12) for \( x \) and subtracting the area of the door before dividing by 56. The student rounds up the answer (8.28) to 9 rolls.

In Part C, the following calculations are done correctly: finding the perimeter in feet and converting to yards, finding the number of rolls of border and its cost, subtracting the cost of the border from $350 and dividing the difference by 9.

An error in computing one of these amounts is allowed provided the process is correct. Other answers based on this error will be considered correct if the correct procedure is used.

2 Points  The response has correct answers for all three parts, but the explanations may be incomplete or are not clearly stated, leaving the reader to make inferences. OR Two parts of the problem have correct values and are completely explained; the third part may be missing, have an incorrect value due to a process error, or is missing an explanation. OR The response has minor computational errors throughout, although the procedures used are correct.

1 Point One part of the question is correctly done with an adequate explanation. OR The response contains two correct values without explanation or with partial explanations. OR The response overall shows some understanding of the mathematical concepts, but contains major errors. OR The response contains three correct answers, but supporting work and/or explanations are missing or incorrect.

0 Points  The response shows an insufficient understanding of the problem’s essential mathematical concepts. The procedures, if any, contain major errors. There may be no explanation of the solution or the reader may not be able to understand the explanation. The reader may not be able to understand how and why decisions were made.
The response shows an insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
Score Point 0
The response shows an insufficient understanding of the problem’s essential mathematical concepts. The procedures contain major errors.
Part A:
Being that \( x \) equals the width of the room, an equation in terms of \( x \) to determine the total surface area of the playroom's walls would be \( 4(1.5x \times 8) \). Because the length is \( 1\frac{1}{2} \) times the width of the room and the width which is also the height equal \( 8 \), then the area of each wall is \( 1.5 \times 8 \) and since there are \( 4 \) walls in the room, then I multiplied the equation by \( 4 \). That's how I got \( 4(1.5 \times 8) = 4(12) = 48 \).

Part B:
Each wall is \( 48 \) ft and each roll covers approximately \( 50 \) ft. One of the walls has a door which measures \( 3.5 \) ft wide by \( 6.5 \) ft high and will not need wall paper. 3 walls will need \( 48 \) ft of wall paper and one wall will need \( 31.75 \) ft of wall paper. Since each roll has \( 50 \) ft of wall paper, the Sheila is going to need \( 4 \) rolls to cover her walls and she will have \( 48.25 \) ft of wall paper left.

Part C:
Two of the walls in the room equal \( 12 \) ft, which is \( 4 \) yards. The other two walls are \( 8 \) ft each which equal \( 2 \) yards, and at \( \frac{3}{4} \) each, Sheila would have to buy a roll of border for \( $19.94 \) and she will have about \( 6 \) yards of the border left. Since Sheila has a budget of \( $350.00 \) and spent \( $39.94 \) on the border, then the
The response overall shows some understanding of the problem’s essential mathematical concepts but contains major errors.
A) Let \( x \) be the width of the room. Write and simplify an equation in terms of \( x \) to determine the total surface area (\( S \)) of the playroom's walls.

B) The playroom is 12 feet wide. Each roll of wallpaper will cover approximately 56 square feet. The wallpaper has no pattern; therefore, it is not necessary to allow for matching patterns when calculating the amount of wallpaper to purchase. What is the \textbf{least} number of rolls Sheila will need to paper her room? Show your work.

C) Sheila plans to spend no more than $350.00 (ignore tax) on the wallpaper and border. The border costs $19.97 per 10-yard roll. The border will be pasted onto the wallpaper at the top of each wall with no overlap. Explain how to determine the most expensive wallpaper Sheila can buy while staying within her budget.

\[ x = 12 \text{ feet} \]

The area of the whole playroom is \( 463 \) and \( \frac{3}{4} \) square feet.

So, Sheila will need 9 rolls of wallpaper, \( \frac{56}{4} \).

She will have 504 square feet of wallpaper, but she will have 40 square feet left.

She will need 60 yards of border paper, so that makes 6 yards.

The border paper is going to cost her \( \frac{300}{10} \) per yard, or \$19.82.

She has \( 230.18 \) to spend on wallpaper.

I estimate that each roll of wallpaper costs about \$5.58.

She can go over \$5.58 on wallpaper in order to stay in her budget.

Materials/Resources:
- Calculator
- HSPA Mathematics Reference Sheet

Techniques for PAT Scoring:
See Mathematics Item-Specific Rubric C1006-008R for this PAT.

Score Point 1
The student does not find an equation for part A; however, the student does find that the area of the room is 463.75 sq ft and that Sheila needs 9 rolls of wallpaper.
π ≈ 3.14 or \(\frac{22}{7}\)

**Circle**

Area = \(\pi r^2\)

Circumference = \(2\pi r\)

**Rectangle**

Area = \(lw\)

Perimeter = \(2(l + w)\)

**Parallelogram**

Area = \(bh\)

**Pythagorean Formula**

\[c^2 = a^2 + b^2\]

**Trapezoid**

Area = \(\frac{1}{2}(b_1 + b_2)h\)

**Triangle**

Area = \(\frac{1}{2}bh\)

**Cone**

Volume = \(\frac{1}{3}\pi r^2 h\)

**Sphere**

Volume = \(\frac{4}{3}\pi r^3\)

**Cylinder**

Volume = \(\pi r^2 h\)

The sum of the measures of the interior angles of a triangle = 180°

The measure of a circle is 360° or \(2\pi\) radians

Distance = rate \times time  
Interest = principal \times rate \times time

**Compound Interest Formula:** \(A = p \left(1 + \frac{r}{k}\right)^{kt}\)

\(A\) = amount after \(t\) years;  
\(p\) = principal;  
\(r\) = annual interest rate;  
\(t\) = number of years;  
\(k\) = number of times compounded per year

The number of combinations of \(n\) elements taken \(r\) at a time is given by \(\frac{n!}{(n-r)!r!}\)

The number of permutations of \(n\) elements taken \(r\) at a time is given by \(\frac{n!}{(n-r)!}\)

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