Guidelines for Establishing GPS-Derived Orthometric Heights
(Standards: 2 cm and 5 cm)

Version 1.4

David B. Zilkoski
Edward E. Carlson
Curtis L. Smith

National Geodetic Survey
1315 East-West Highway
Silver Spring, Maryland 20910
(301) 713-3191

October 2005
CONTENTS

Preface .................................................. 3
Introduction .............................................. 3
The 3-4-5 System .......................................... 6
Three Basic Rules ........................................ 6
Four Basic Requirements ................................ 6
Five Basic Procedures ................................... 7
Submission of Data to the National Geodetic Survey .............. 8
Guideline Updates ....................................... 9
References .............................................. 11
Appendix A - Definitions ............................... 13
Appendix B - GPS Ellipsoid Height Hierarchy and Basic Requirements ... 14
Appendix C - Illustration of GPS-Derived Orthometric Height Guidelines
Using a Sample Project .................................. 17
Appendix D - Ten Minimum Steps Required to Estimate and Evaluate a
GPS-Derived Orthometric Height Project ....................... 25
GUIDELINES FOR ESTABLISHING GPS-DERIVED ORTHOMETRIC HEIGHTS

[Standards:  2 cm and 5 cm]
Version 1.4

Preface

In November 1997, the National Geodetic Survey (NGS) published guidelines for performing Global Positioning System (GPS) surveys intended to achieve ellipsoid height network accuracies of 5 cm and ellipsoid height local accuracies of 2 cm or 5 cm, both at the 95 percent confidence level (Zilkoski et al. 1997).

Definitions: Local accuracy - a value that represents the uncertainty in the coordinates of the point relative to the coordinates of other local, directly connected, adjacent points.

Network accuracy - a value that represents the uncertainty in the coordinates of the point with respect to the geodetic datum.

See Appendix A for additional information on local and network ellipsoid height accuracies, and Appendix B for information on the basic requirements for 2-cm ellipsoid height standards.

NGS developed the following guidelines for performing GPS surveys intended to achieve orthometric height network accuracies of 5 cm and orthometric height local accuracies of 2 cm or 5 cm, both at the 95 percent confidence level. These guidelines were developed in partnership with federal, state, and local government agencies, academia, and private surveyors.

Following these guidelines should produce intended accuracies; they were designed to assist in establishing vertical control networks. Additionally, some of the guidelines may be relaxed in the future. Also note the intended accuracies may be achieved without strict adherence to these guidelines. The base line comparison and adjustment results provide proof of accomplishment, or the lack thereof. Detailed discussion of field and office procedures should be documented in the project report, to be provided with data submissions to NGS. This will enable NGS to analyze procedures and results that may merit modifications to these guidelines.

Introduction

Since 1983, NGS has performed control survey projects in the United States using GPS surveying techniques. Analysis of that survey data has shown that GPS can be used to establish precise relative positions in a three-dimensional, Earth-centered coordinate system. GPS carrier-phase measurements are used to determine vector base lines in space, where the components of the base line are expressed in terms of Cartesian coordinate differences (delta x, y, and z) (Remondi 1984). The vector base lines can be converted to distance, azimuth, and ellipsoidal height difference (dh), relative to a defined reference ellipsoid.

Can the accuracies achieved for these **orthometric** height differences now provide a viable alternative to classical geodetic leveling techniques? With the completion of the general adjustment of the North American Vertical Datum of 1988 (NAVD 88) (Zilkoski et al. 1992), computation of an accurate national high-resolution geoid model (currently GEOID03 with new models under development) (Roman et al. 2004), and publication of NGS' Guidelines for Establishing GPS-Derived Ellipsoid Heights (Standards: 2 cm and 5 cm) (Zilkoski et al. 1997), the answer is yes! GPS-derived orthometric heights can provide a viable alternative to classical geodetic leveling techniques for many applications.

Orthometric heights \( H \) are referenced to an equipotential reference surface, e.g., the geoid. The orthometric height of a point on the Earth's surface is the distance from the geoidal reference surface to the point, measured along the plumb line, normal to the geoid. Ellipsoid heights \( h \) are referenced to a reference ellipsoid. The ellipsoid height of a point is the distance from the reference ellipsoid to the point, measured along the line which is normal to the ellipsoid. At the same point on the surface of the earth, the difference between an ellipsoid height and an orthometric height is defined as the geoid height \( N \).

Several error sources that affect the accuracy of orthometric, ellipsoid, and geoid height values are generally common to points near each other. Because these error sources are in common, the uncertainty of height differences between nearby points is significantly smaller than the uncertainty of the absolute heights of each point.

Orthometric height differences \( \Delta H \) can be obtained from ellipsoid height differences, by subtracting the geoid height differences \( \Delta N \):

\[
\Delta H = \Delta h - \Delta N.
\]

Adhering to NGS' earlier guidelines, ellipsoid height differences \( \Delta h \) over short
base lines, i.e., not more than 10 km, can now be determined to better than +/- 2 cm (with 2-sigma uncertainty) from GPS phase measurements. This is possible because of the availability of a greater number of satellites; more accurate satellite orbits; full-wavelength, dual-frequency carrier phase data; improved antenna designs; more continuously operating reference stations (CORS) serving as geodetic control; and improved data processing techniques. Also, the GPS-derived ellipsoid height guidelines (Zilkoski et al. 1997) were intentionally designed to produce ellipsoid heights better than 2 cm, i.e. about 1.4 cm, so they could also be used to generate 2 cm GPS-derived orthometric heights. Each base line must be repeated, and the ellipsoid height differences between the two points of the base line must agree to within 2 cm of each other. They must have been obtained on two separate days, at different times of the day. By following these local accuracy requirements, final GPS-derived ellipsoid heights, better than 2 cm local accuracy, at the 2-sigma level, can be achieved. The requirement that spacing between local network stations cannot exceed 10 km helps keep the relative error in geoid height small; i.e., typically less than 0.5 cm. Adding in small error for uncertainty of geoid height difference and controlling remaining systematic differences between the three height systems will typically produce a GPS-derived orthometric height with 2-sigma uncertainties, with +/- 2 cm local accuracy.

Geoid height differences can be determined (in select areas nationally) with uncertainties that are typically better than 1 cm for distances up to 20 km, and less than 2-3 cm for distances between 20 and 50 km (Zilkoski and D’Onofrio 1996, and Henning et al. 1999). Small values for the differential geoid height uncertainties have been demonstrated in tests in several regions of the United States. Larger uncertainties can be expected in other areas, depending on the density of the observed gravity network, and uncertainties in the determination of observed and interpolated gravity anomalies. Determining uncertainties in geoid height differences, through a comparison of leveled and GPS height differences, depends on the error sources in the leveled and GPS height differences. The NGS guidelines allow the user to know the GPS height errors and allowable tolerances (Zilkoski et al. 1997). Analysis of the leveling and geoid error is more complex and will need to be addressed on a case-by-case basis. With a high resolution geoid model, currently GEOID03, and valid NAVD 88 heights, surveys in most non-mountainous regions of the conterminous United States will produce several-centimeter-level results, when these GPS-derived orthometric height guidelines are followed. In mountainous regions of the United States, where the geoid and height uncertainties are typically larger, and published NAVD 88 vertical control stations are usually sparse, the user should contact NGS for assistance in the design and analysis of their GPS project (See contact information at the end of the document).

NOTE: The term “user” in this document refers to a person who uses GPS surveying techniques and/or analyzes GPS data to determine height and position information.

When high-accuracy field procedures for precise geodetic leveling are used, orthometric height differences can be computed with an uncertainty of less than 1 cm, over a 50-kilometer distance. Depending on the accuracy requirements, GPS surveys and current high-resolution geoid models can be used instead of classical leveling methods. In the past, the primary limiting factor was the accuracy of estimating geoid height differences; with the computation of the latest national high-resolution geoid model (currently GEOID03), and the development of the 2- and 5-cm guidelines for estimating GPS-derived ellipsoid heights (Zilkoski et al. 1997), the limiting factor is...
often the lack of valid NAVD 88 orthometric heights available for vertical control. Strategically, occupying bench marks with GPS and valid NAVD 88 height values is critical to detecting, reducing, and/or eliminating blunders and systematic errors between the three height systems.

The 3-4-5 System

There are three basic rules, four control requirements, and five procedures necessary for estimating GPS-derived orthometric heights. This document describes their requirements, in order to meet 2- or 5-cm standards, and does so in brief format. Detailed explanations can be found in the referenced reports. Appendix C contains a brief description of the 3-4-5 system using a sample project.

Three Basic Rules

Rule 1: Follow NGS’ guidelines for establishing GPS-derived ellipsoid heights when performing the GPS survey (Zilkoski et al. 1997). Follow the specific guidelines for desired orthometric heights. For example, use the guidelines for achieving 2 cm GPS-derived orthometric heights for 2 cm ellipsoid heights, and the guidelines for 5 cm GPS-derived orthometric heights for 5 cm ellipsoid heights.

Rule 2: Use NGS’ latest national geoid model, currently GEOID03, when computing GPS-derived orthometric heights (Roman et al 2004).

Rule 3: Use the latest National Vertical Datum, i.e., NAVD 88, height values to control the project’s adjusted orthometric heights (Zilkoski, et al, 1992).

Four Control Requirements

Requirement 1: Occupy stations with valid NAVD 88 orthometric heights. Stations should be evenly distributed throughout project. A previously determined GPS-derived orthometric height, accurate to 2 cm, IS considered a valid’ NAVD 88 height if it is in the National Spatial Reference System (NSRS), i.e., the NGS database. In these requirements, a ‘valid NAVD 88 bench mark’ includes vertical control that has been leveled and/or has an orthometric height valid to 2-cm accuracy.

Requirement 2: For project areas less than 20 km on a side, surround project with valid NAVD 88 bench marks, i.e., minimum number of stations is four, one in each corner of project.

NOTE: The project area may need to be enlarged to occupy enough bench marks, even if the area extends beyond the original area of interest.

Requirement 3: For project areas greater than 20 km on a side, keep distances between valid GPS-occupied NAVD 88 bench marks to less than 20 km.

NOTE: When possible, occupy extra NAVD 88 bench marks in case some bench mark heights are inconsistent.
Requirement 4: For projects located in mountainous regions, occupy valid bench marks that are at both the lowest elevation and the highest elevation of the area, even if the distance is less than 20 km. Consider adding additional bench marks to get a good range of elevation change.

NOTE: Valid NAVD 88 height values include, but are not limited to, bench marks in the NSRS which have not moved since their heights were last determined (that is, if they have been re-leveled and their latest observation is found to match the previous observation).

Five Basic Procedures

Procedure 1: Perform a 3-D minimum-constraint least squares adjustment of the GPS survey project, i.e., constrain the latitude and longitude of one NSRS control station, and one orthometric height value.

Procedure 2: Detect and remove all data outliers, i.e., high residuals, for a base line using the results from the adjustment in procedure 1 above.

Note: The user should repeat procedures 1 and 2 until all data outliers are removed.

Procedure 3: Compute differences between the set of GPS-derived orthometric heights from the minimum constraint adjustment (using the latest national geoid model, currently GEOID03) from procedure 2 above and published NAVD 88 orthometric heights.

Procedure 4: Using the results from procedure 3 above, determine which vertical NSRS control stations have valid NAVD 88 height values. This is the most important step of the process. Determining which bench marks have valid heights is critical to computing accurate GPS-derived orthometric heights. All differences between GPS observations on valid bench marks need to agree within 2 cm for 2-cm surveys and 5 cm for 5-cm surveys.

NOTE: For most small area projects, (e.g., 20 km by 20 km, in the conterminous United States) using NGS’ latest geoid model should produce satisfactory results (see Hennings et. al, 1998).

Large areas (i.e. 50 km by 50 km) may have a systematic tilt -- this tilt can be accounted for in the final constrained adjustment, with NAVD 88 vertical NSRS control stations occupied with GPS, every 20 km. However, for detecting NAVD 88 height outliers, the user should estimate local systematic differences between GPS-derived heights and leveling-derived heights, by solving and removing this systematic difference. [See Vincenty (1987) and Zilkoski (1993). Documentation of the process of computing and removing a systematic tilt is in progress.

Procedure 5: Using the results from procedure 4 above, perform a constrained orthometric height adjustment by fixing the latitude and longitude of one NSRS control station and all valid NAVD 88 heights.

The user should always ensure the final set of heights is not overly distorted by the adjustment process. This should not occur if the procedures outlined above are followed.
To check the influence of additional constraints on the network, compute the differences between the fully-constrained set of GPS-derived orthometric heights from procedure 5, and the minimally constrained set of heights from procedure 2. The comparison of the two sets of orthometric height differences between neighboring stations should not have large (i.e. > 1 cm) differences (see Henning et al. 1998). If these differences exceed 2 cm, it is possible that an incorrect or invalid vertical control value was held fixed.

NGS has prepared several reports that describe these procedures in more detail (Zilkoski and Hothem, 1989; Zilkoski, 1990a; Zilkoski, 1990b; Zilkoski, 1993; and Henning, et. al, 1998). These reports are available from NGS’ Web site at [www.ngs.noaa.gov/PUBS_LIB/pub_index.html](http://www.ngs.noaa.gov/PUBS_LIB/pub_index.html).

Due to improvements in high resolution geoid models, implementation of the full constellation of GPS, completion of the NAVD 88 project, improvements in GPS equipment and processing software, and the development of guidelines for estimating GPS-derived ellipsoid heights, the steps outlined in the above reports need to be considered only when a problem is detected during the performance of the five procedures. However, the reports, although slightly outdated (because of improvements in geoid models and technology) should provide the necessary information for the user to understand how to perform the five procedures stated in these guidelines. In particular, the report titled “NGS/Caltrans San Diego GPS-Derived Orthometric Height Cooperative Project” (Zilkoski, 1993) demonstrates the minimum steps required to estimate and evaluate a GPS-derived orthometric height project. Today, the ten steps are simplified into five procedures, but they may still need to be considered when doing some projects. Appendix D contains a list of the ten steps outlined in the San Diego GPS Project report and Appendix C illustrates application of the five procedures by using a sample project.

**Submission of Data to the National Geodetic Survey**

"Input Formats and Specifications of the National Geodetic Survey (NGS) Data Base," commonly called the "Blue Book," is a contributor’s guide for preparing and submitting geodetic data for incorporation into NGS' data base. Survey data entered into NGS' data base become part of the National Spatial Reference System (NSRS). The guide has three volumes -- Volume I covers classical horizontal geodetic and Global Positioning System (GPS) data; Volume II covers vertical geodetic data; and Volume III covers gravity data.

Survey data submitted to NGS for incorporation into the NSRS should be properly formatted and adhere to the guidelines outlined in the latest version of the “Blue Book.”

The “Blue Book,” and most of the documents referenced herein, may be obtained from the NGS web site at [http://www.ngs.noaa.gov/FGCS/BlueBook/](http://www.ngs.noaa.gov/FGCS/BlueBook/) or

NOAA, National Geodetic Survey, N/NGS12
1315 East-West Highway, Station 9202
Silver Spring, MD 20910-3282
Telephone: (301) 713-3242; Fax: (301) 713-4172
Monday through Friday, 7:00 a.m. - 4:30 p.m. Eastern Time
The formats and specifications of the “Blue Book” are consistent with the aims of the Executive Office of the President, Office of Management and Budget's (OMB) Circular A-16, as revised in 1990. A major goal of the circular, which is titled "Coordination of Surveying, Mapping, and Related Spatial Data Activities," is to develop a national spatial data infrastructure with the involvement of Federal, state, and local governments, and the private sector. This multilevel national information resource, united by standards and criteria established by the Federal Geodetic Control Subcommittee (FGCS) of the Federal Geographic Data Committee (FGDC) will enable the sharing and efficient transfer of geospatial data submitted to populate the NSRS.

**Data Submission to NGS**

1. A project accession number will be assigned when the project is received by NGS.

2. A project report and the data elements listed in Appendix L of "Input Formats and Specifications of the NGS Data Base" must be transmitted to NGS. Quality checks for conformance to NGS format standards are done using required software programs. See website page: www.ngs.noaa.gov/PC_PROD/ADJUST/.

3. Latitude, longitude, and ellipsoid heights, as well as X, Y, and Z coordinates, shall be provided in both NAD 83 and ITRF coordinate systems. GPS-derived orthometric heights shall be provided in NAVD 88.

**Guideline Updates**

These Guidelines are likely to be updated as results from future projects and other (modified) procedures are reviewed. There may be other procedures that will also achieve the standards. The contributor should note which procedures in this document were not followed and note how errors and systematic biases were detected, reduced, or eliminated by the alternate procedure. NGS welcomes the opportunity to examine alternate procedures and supporting data that demonstrate the ability to achieve the accuracy standards stated in this document. If you have such data or would like to discuss alternative procedures, please contact Dave Zilkoski, Edward Carlson, or Curt Smith.

**Contact Information**

Dave Zilkoski  
Email: Dave.Zilkoski@noaa.gov  
Tel: 301-713-3222

Edward Carlson  
Email: Ed.Carlson@noaa.gov  
Tel: 808-532-3205

Curt Smith  
Email: Curt.Smith@noaa.gov  
Tel: 208-332-7197
State Geodetic Advisors - Many states have a geodetic advisor whose objective it is to enable others to contribute data to and utilize data of the NSRS. Because they may be aware of areas within their jurisdiction with problematic vertical control, it can be productive to engage in discussions with that advisor. To locate an advisor near your area of interest, refer to www.ngs.noaa.gov/ADVISORS/AdvisorsIndex.shtml.

Additional Height Modernization information can be accessed on NGS’ Height Modernization web page at www.ngs.noaa.gov/heightmod/.
REFERENCES


Zilkoski, D., J. D. D’Onofrio, and S. Frakes, 1997: Guidelines for Establishing GPS-Derived Ellipsoid Heights (Standards: 2 cm and 5 cm), Version 4.3, NOAA Technical Memorandum NOS NGS-58, National Geodetic Survey Information Center, Silver Spring, MD 20910.


Appendix A: Definitions

Accuracy

**Local Accuracy** - The local accuracy of a NSRS point is a value that represents the uncertainty in the coordinates of the point relative to the coordinates of other local, directly connected, adjacent points at the 95-percent confidence level. The reported local accuracy is an approximate average of the individual local accuracy values between this NSRS point and other observed local points used to establish the coordinates of the NSRS point (Geospatial Positioning Accuracy Standards, Federal Geodetic Control Subcommittee, 1998).

**Network Accuracy** - The network accuracy of a point is a value that represents the uncertainty in the coordinates of the point with respect to the geodetic datum at the 95-percent confidence level. For NSRS network accuracy classification, the datum is considered to be best supported by NGS. By this definition, the local and network accuracy values at CORS sites are considered to be infinitesimal, i.e., to approach zero (Geospatial Positioning Accuracy Standards, Federal Geodetic Control Subcommittee, 1998).

Stations

**NSRS Stations** (~75 km spacing) High accuracy NSRS three-dimensional stations (CORS, FBN, HARN stations) that surround the project area in at least three different quadrants. These stations relate the local network to the NSRS through simultaneous observations with primary base stations. They provide the network accuracy. They may be newly established stations in the survey project if specifications and procedures are used to establish them. These procedures are not covered in this document. Please contact NGS for additional information.

**Primary Base Stations** (~40 km spacing) Stations evenly distributed that surround the local network. These stations relate the local network to NSRS to the 5-cm, or better, standard through simultaneous observations with NSRS control stations. They can be newly established stations and be part of the local network.

**Secondary Base Stations** (~15 km) Stations evenly distributed throughout the local network that ensure that the local network does not contain a significant medium wavelength (20-30 km) ellipsoid height error through simultaneous observations with primary base stations. These stations may be newly established stations and are part of the local network. They are located between Primary Base Stations.

**Local Network Stations** (<10 km) Stations that are not NSRS, primary base, or secondary base stations but are part of the local network. They provide the local accuracy standard through simultaneous observations between adjacent stations.
Appendix B: GPS Ellipsoid Height Hierarchy and Basic Requirements

Basic Requirements:

- 5 hour sessions for 3 days
- Spacing between primary base stations cannot exceed 40 km.
- Each primary base station must be connected to at least its nearest primary base station neighbor and nearest NSRS control station.
- Primary base stations must be traceable back to two NSRS control stations along independent paths; i.e, base lines Primary1-NSRS1 and Primary1-NSRS2, or Primary1-Primary2 plus Primary2-NSRS3.
Basic Requirements:

- 30 minute sessions/2 days/significantly different satellite geometry.

**NOTE:** Thirty minute sessions are the minimum requirement. In some locations, due to abnormal atmospheric conditions, poor satellite geometry, and local multipath effects, it may be necessary to collect more than 30 minutes of data to meet the 2-cm repeat base line requirement.

- Spacing between secondary base stations (or between primary and secondary base stations) cannot exceed 15 km.

- All base stations (primary and secondary) must be connected to at least its two nearest primary or secondary base station neighbors.

- Secondary base stations must be traceable back to two primary or NSRS base stations along independent paths; i.e., Secondary1-NSRS1 and Secondary1-Secondary3 plus Secondary3-Primary1.

- Secondary base stations need not be established in small area surveys.
Basic Requirements:

- Thirty (30) minute sessions / 2 days / significantly different satellite geometry (PDOP)

**NOTE:** Thirty minute sessions are the minimum requirement. In some locations, due to abnormal atmospheric conditions, poor satellite geometry, and local multipath effects, it may be necessary to collect more than 30 minutes of data to meet the 2-cm repeat base line requirement.

- Spacing between local network stations (or between base stations and local network stations) cannot exceed 10 km.

- All local network stations must be connected to at least its two nearest neighbors.

- Local network stations must be traceable back to 2 primary base stations along independent paths.
Using GPS, Geoid99, and NGS Guidelines to Obtain Reliable, Accurate, Orthometric Heights in Support of Photogrammetric and Surveying Project in Baltimore County, Maryland

Three Basic Rules

The project fulfilled the three basic rules by using:

1. NGS’ guidelines for establishing GPS-derived ellipsoid heights (Standards: 2 cm and 5 cm);
2. The latest national geoid model at the time of the survey (currently GEOID03); and,
3. Stations with valid orthometric heights in NAVD 88, the latest national vertical datum.

Four Basic Control Requirements

1. Occupy stations with valid NAVD 88 orthometric heights (stations should be evenly distributed throughout project).

2. For project areas less than 20 km on a side, surround project with NAVD 88 bench marks, i.e., the minimum number of stations is four; one in each corner of project.

3. For project areas greater than 20 km on a side, keep distances between GPS-occupied valid NAVD 88 bench marks to less than 20 km.

4. For projects located in mountainous regions, occupy valid bench marks at the base and the summit of the mountains, even if the distance between them is less than 20 km. [If you cannot occupy the summit, then occupy the highest point closest to the summit.]

---

BCR1: Stations shown as bench marks or bench marks within circles have valid NAVD 88 heights. These stations are evenly distributed throughout the project area.

BCR2: This requirement is not applicable because the project is greater than 20 km on a side. [If the project was less than 20 km on a side, then there should be at least four stations with NAVD 88 heights.]

BCR3: Circled bench marks are mandatory. The analysis must determine that the bench marks have valid NAVD 88 heights. Other bench marks can be substituted but the user must adhere to the 20 km requirement.

NOTE: Valid NAVD 88 height values include, but are not limited to, the following: bench marks which have not moved since their heights were last determined, were not misidentified, and are consistent with NAVD 88.
BRC4: This requirement is not applicable because project is not in a mountainous region. [If the project was in a mountainous region then bench marks should be located at the base and summit of the mountainous region.]

Five Basic Adjustment Procedures

BAP1: Perform a 3-D minimum-constraint least squares adjustment of the GPS survey project, i.e., constrain one latitude, one longitude, and one orthometric height value.

BAP2: Using the results from the adjustment in procedure 1 above, detect and remove all data outliers. The user should repeat procedures 1 and 2 until all data outliers are removed.

Note: After performing the minimum constraint adjustment, the user should plot the ellipsoid height residuals (or dU residuals) and investigate all residuals greater than 2 cm.

Free Ellipsoid Height Residual by Baseline Length

455 Baselines - 2.6% above 3 cm

Note that the station pairs that have large residuals, i.e., greater than 2.5 cm, also have large repeat base line differences. The NGS guidelines for estimating GPS-derived ellipsoid heights require the user to reobserve these base lines. Following NGS guidelines provides enough redundancy for the adjustment process to detect outliers and apply the residual on the appropriate observation, i.e., the bad vector.
BAP 3: Compute differences between the set of GPS-derived orthometric heights from the minimum constraint adjustment from procedure 2 above and published NAVD 88 bench marks.
Note: All height differences are under 5 cm and most are less than 2 cm. Almost all relative height differences between adjacent station pairs are less than 2 cm. However, most of the height differences appear to be positive relative to the southwest corner of the project.

BAP 4: Using the results from procedure 3 above, determine which bench marks have valid NAVD 88 height values. All differences between valid benchmark marks need to agree within 2 cm for 2-cm surveys and 5 cm for 5-cm surveys.
Note: To detect and remove any systematic trend, a tilted plane is best fit to the height differences (Vincenty 1987, Zilkoski and Hothem 1989). After a trend has been removed, all the differences are less than +/- 2 cm except for one outlier and almost all relative differences between adjacent stations are less than 2 cm.

BAP 5: Using the results from procedure 4 above, perform a constrained adjustment fixing one latitude and one longitude value and all valid NAVD 88 height values.
Note: After rejecting the largest height difference (~2.4 cm), of all the closely spaced (less than 10 km) station pairs, only three are greater than 2 cm, one is greater than 2.5 cm and none are greater than 3 cm. [Not shown]

There are 25 stations with both GPS heights and NAVD 88 heights. This makes 300 unique comparisons. Of these comparisons, 59 are greater than 2.0 cm but only 34 are greater than 2.1 cm, and none are greater than 2.5 cm. [Not shown]
Baltimore County GPS Project
GPS-Derived Orthometric Heights minus NAVD 88 Values
(All Unique Height Difference Combinations - Sample Size = 300)
APPENDIX D: Ten Minimum Steps Required to Estimate and Evaluate a GPS-Derived Orthometric Height Project

These steps are documented in the report titled “Minimum Steps Required When Estimating GPS-Derived Orthometric Heights,” Proceedings of the GIS/LIS ’90 Fall Convention, Anaheim, California, November 7-10.

The minimum steps required when analyzing GPS-derived orthometric heights are listed below.

1. During the project’s planning stage, perform a detailed analysis of the geoid in the area of the survey in order to determine if additional gravity and/or leveling data are required to adequately estimate the slope of the geoid and changes in slope.

2. During the project’s planning stage, perform a detailed study of the leveling network in the area, i.e., plot all leveling lines, note the age of leveling, determine if bench marks can be occupied by GPS receivers, etc.

3. Perform a 3-D minimum constraint least squares adjustment of the GPS data and compare GPS-derived coordinates with results of higher-order surveys.

4. Use the best available geoid model and compare adjusted GPS-derived orthometric height differences obtained from step 3 with leveling-derived orthometric height differences.

5. Detect and remove all data outliers determined in steps 3 and 4.

6. Analyze the local geoid in detail.
   a. Plot the modeled geoid heights in the area.
   b. Plot the estimated slope of the geoid using differences between GPS derived ellipsoid height differences and leveling-derived orthometric height differences \( (dN = dh - dH) \) obtained in step 4.


8. Compare adjusted GPS-derived orthometric height differences from step 7 with leveling-derived orthometric height differences to determine scale and rotation parameters.

9. Compare GPS-derived orthometric heights by performing a 3-dimensional least squares adjustment holding fixed all appropriate orthometric height values of published bench marks (and approximate GPS-derived coordinates computed from higher-order surveys and solving for appropriate scale and rotation parameters.

10. Use the results from steps 1 through 9 to document the estimated accuracy of the GPS-derived orthometric heights.